Towards Modeling of Nonlinear Laser-Plasma Interactions with Hydrocodes: the Thick Rays Model

A. Colaïtis, G. Duchateau, Ph. Nicolaï and V. T. Tikhonchuk

Univ. Bordeaux, CEA, CNRS, Centre Lasers Intenses et Applications, 351 cours de la libération, Talence, France

Introduction

The CELIA laboratory is studying direct drive schemes for Inertial Confinement Fusion (ICF). Theoretical and numerical studies are supported by experiments in laser facilities worldwide. In particular, the shock ignition scheme is studied within a european collaboration and will be tested on the French MégaJoule laser system. Shock ignition requires high laser intensities, resulting in nonlinear laser-plasma interactions where the optical wave couples to electron and ion plasma waves, notably resulting in Stimulated Raman Scattering, Stimulated Brillouin Scattering and Cross-Beam Energy Transfer. These nonlinear processes are commonly studied at microscopic scales but often omitted in larger-scale hydrodynamical codes. The importance of such effects has been highlighted by recent experiments on NIF and OMEGA and these must be included in large-scale models. The description of nonlinear laser plasma interactions requires the knowledge of the wave's electric field in the plasma. Radiative-hydrodynamic codes usually rely on Ray-Tracing models (RT) [1] which describe needle-like rays. This approach neglects diffraction effects and hence does not allow to compute precisely the intensity of the laser beam. We have recently developed an alternative to RT models adapted from a thick-ray description of a wavefield (Paraxial Complex Geometrical Optics (PCGO)) commonly used in optics, geophysics, accoustics, radio physics and plasma physics [2]. Our model has been reformulated for dense non-uniform plasmas and validated against several academic text cases and against the RT model implemented in the CHIC hydrocode. It is found that diffraction effects impact energy deposition in ICF plasmas.

Basic Equations of Geometrical Optics

Both the Ray-Tracing and PCGO methods rely on Geometrical Optics (GO) principles. The starting point of GO is the Helmholtz equation. For a monochromatic wave, the scalar wave field $u(t, \boldsymbol{r})$ takes the form:

$$\Delta u(\omega, \boldsymbol{r}) + k_0^2 n^2(\omega, \boldsymbol{r}) u(\omega, \boldsymbol{r}) = 0 \tag{1}$$

where $u(\omega, \boldsymbol{r})$ is the electric field amplitude at frequency ω, $\boldsymbol{r}$ is the spatial coordinate, $k_0 = w_0/c$ is the vacuum wave vector and n is the index of refraction of the medium. For a monochromatic wave, the general solution of the Helmholtz equation takes the form of an almost-plane wave:

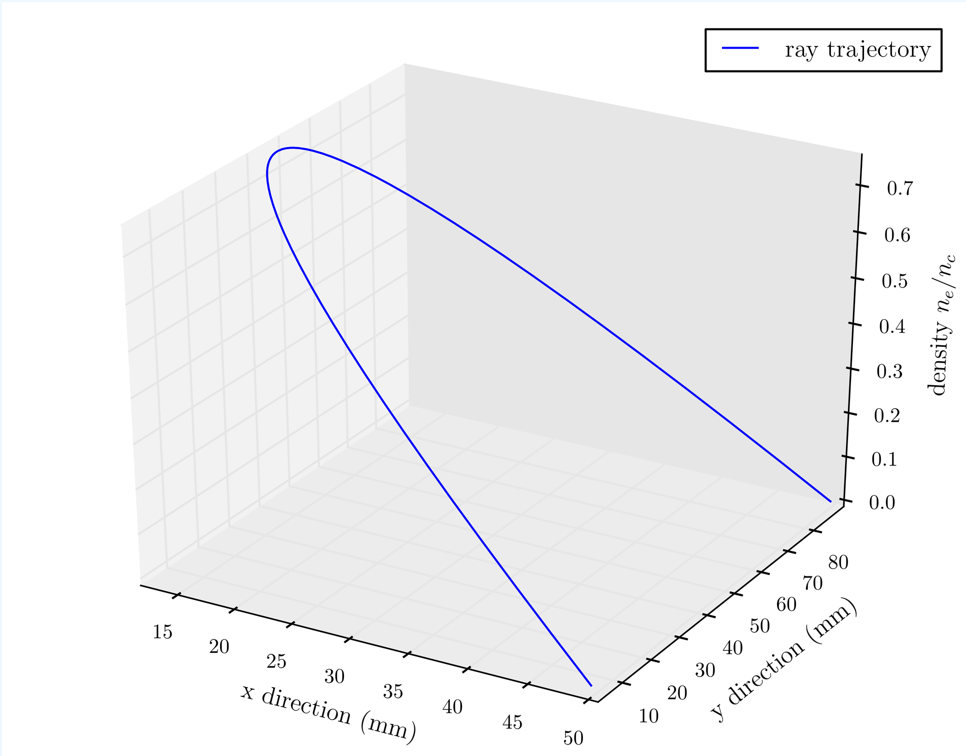
$$u(\boldsymbol{r}) = A(\boldsymbol{r}) \exp[\imath k_0 \psi(\boldsymbol{r})] \tag{2}$$

where $A(\boldsymbol{r})$ is a slowly varying amplitude and $\psi(\boldsymbol{r})$ is the eikonal, or optical path. The scalar field can then be expanded in inverse powers of the vacuum wave number (Debye expansion):

$$A(\boldsymbol{r}) = A_0(\boldsymbol{r}) + \frac{A_1(\boldsymbol{r})}{ik_0} + \frac{A_2(\boldsymbol{r})}{(ik_0)^2} + \dots \tag{3}$$

Substitution of this expression into the Helmholtz equation yields a series of equations at different orders in $1/k_0$. The 0th-order is called eikonal equation and reads:

$$(\nabla \psi)^2 = \epsilon(\boldsymbol{r}) = n^2 \tag{4}$$



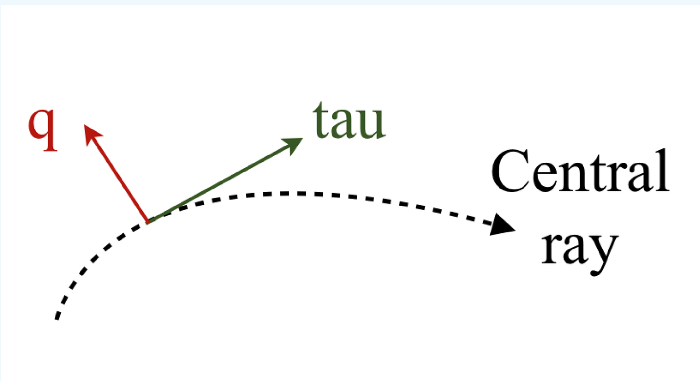
The eikonal equation belongs to the Hamilton-Jacobi variety. Using the characteristics technique yields the trajectory of the rays:

$$\frac{d\boldsymbol{r}}{d\tau} = \boldsymbol{p} \tag{5}$$

$$\frac{d\boldsymbol{p}}{d\tau} = \frac{1}{2} \nabla \epsilon(\boldsymbol{r}) \tag{6}$$

where $\boldsymbol{p} = \nabla \psi$ is assimilated to the momentum of the ray, $\boldsymbol{r}$ is the ray position and τ is linked to the elementary arc length ds by the relation $d\tau = ds/\sqrt{\epsilon'}$. This equation constitutes the basis of RT models and describes point-like particles free-falling in a potential (see left figure).

Paraxial Complex Geometrical Optics



From a given central ray trajectory, computed using standard GO, we define a new coordinate system $\{q_1, q_2, \tau\}$, where q_1 and q_2 are orthogonal to the ray and τ is tangent to it. The eikonal equation (2) and the first order of the Debye expansion (called transport equation) are projected onto this central ray, yielding two equations relating the phase variations with the density field.

Contrary to standard GO, we define a complex laser field phase ψ which accounts for local phase variations around the central ray:

$$\psi(q_1, q_2, \tau) = \overbrace{\psi_c(\tau)}^{\text{RT}} + \overbrace{\tilde{\psi}(q_1, q_2, \tau)}^{\text{Standard PCGO: ray thickness}} + \overbrace{\psi_1(q_1, q_2, \tau)}^{\text{Absorption/Gain model}} \tag{7}$$

$$\text{with} \quad \tilde{\psi}(q_1, q_2, \tau) = \frac{1}{2} B_{ij}(\tau) q_i q_j \tag{8}$$

where $\psi_1(q_1, q_2, \tau)$ is a complex phase perturbation not included in standard PCGO, small compared to $\tilde{\psi}$ and ψ_c, and B is the so-called curvature matrix. Furthermore, we assume that $\psi_1(q_1, q_2, \tau) \equiv \psi_{1c}(\tau)$ and $\psi_{1c}(\tau) \ll \psi_c(\tau)$. The eikonal equation in the new base at order 0 in q_i yields:

$$\left(\frac{\partial \psi_c}{\partial \tau}\right) = \sqrt{\epsilon_c^2} \simeq \epsilon_c' \qquad \left(\frac{\partial \psi_{1c}}{\partial \tau}\right) \simeq \imath \frac{\Im(\epsilon_c^2)}{2\epsilon_c'} = \imath \epsilon_c'' \tag{9}$$

where we have used $\epsilon_c'' \ll \epsilon_c'$. The left equation is the standard equation for the central ray phase in PCGO. The right equation is obtained at the order 1 in ψ and relates the complex phase perturbation to the imaginary part of the relative permittivity, i.e. to the absorption or gain of the medium. At the order 2 in q and using equations (9) we get the Riccati equation for B:

$$B^2 + \frac{\partial B}{\partial \tau} = -\frac{3}{4\epsilon_c'}\left(\frac{\partial \epsilon_c'}{\partial q}\right)^2 + \frac{1}{2}\frac{\partial^2 \epsilon_c'}{\partial q^2} = \alpha(\tau) \tag{10}$$

This equation is identical to the one found in standard PCGO. The expressions for thickness w and curvature radius R of the wave then follow the same form as in standard PCGO:

$$w(\tau) = \sqrt{\frac{2}{k_0 \Im(B(\tau))}} \qquad R(\tau) = \frac{\sqrt{\epsilon_c'}}{\Re(B(\tau))} \tag{11}$$

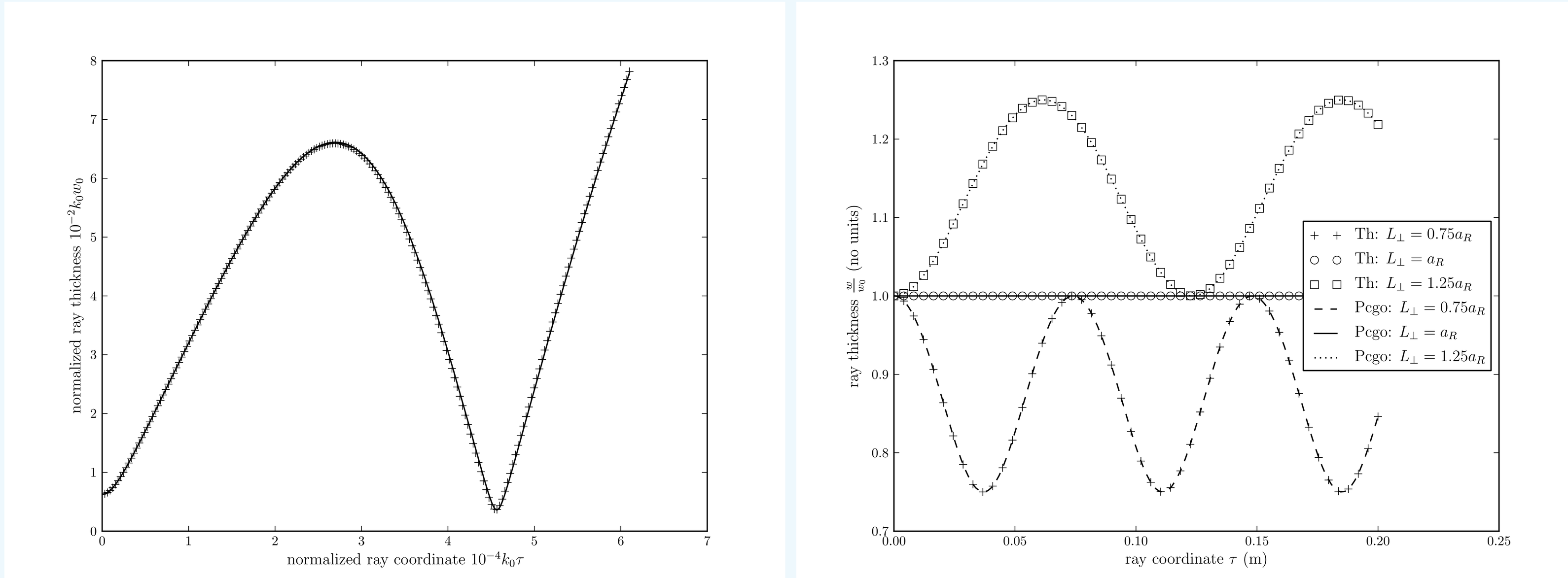
The transport equation projected onto the central ray yields the conservation law for the electric field amplitude. The full form of the electric field then reads:

$$u(q, \tau) = \frac{|\tilde{A}_0(0)|}{(\epsilon_c'(\tau))^{1/4}} \sqrt{\frac{w_0}{w(\tau)}} e^{\imath k_0 \left(\frac{\Re(B)}{2} q^2 - \int_0^\tau \epsilon_c'(\tau) d\tau\right)} e^{-k_0 \left(\frac{\Im(B)}{2} q^2 + \int_0^\tau \epsilon_c''(\tau) d\tau\right)} \tag{12}$$

where $\tilde{A}_0(\tau) = (\epsilon_c'(\tau))^{1/4} A_0(\tau)$ and $w_0 = w(0)$ is the initial Gaussian beam thickness. This equation highlights the decaying factor in the electric field modeled with the contribution of ϵ_c''. The model allows for a consistent description of laser absorption and gives access to the beam thickness and intensity in the plasma. There are several conditions to PCGO applicability, the most restrictive one being that the beam width must be small compared to the characteristic length scale of the inhomogeneities of the plasma L_{ch}, and large compared to the wavelength, i.e. $\lambda_0 \ll w \ll L_{ch}$.

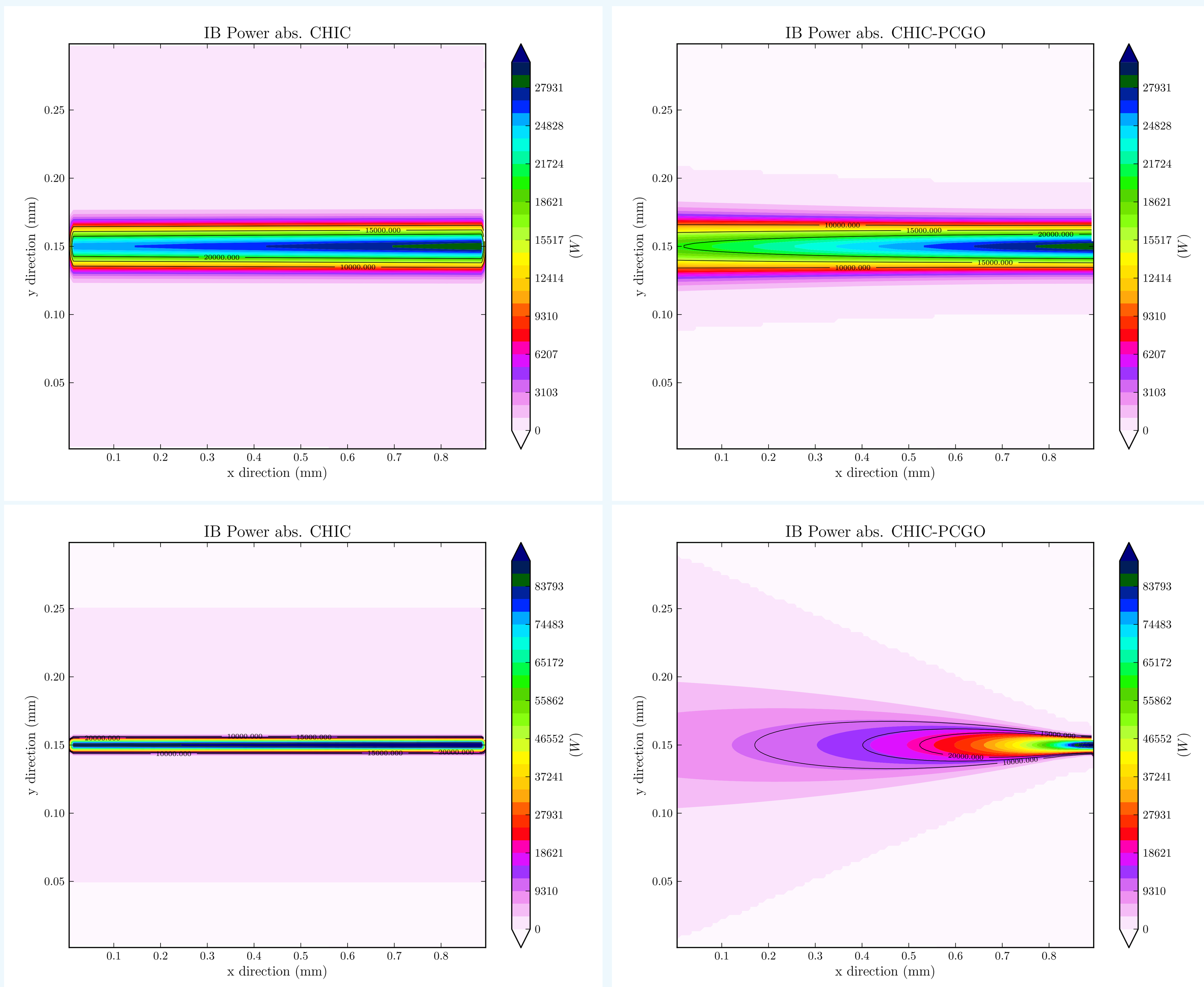
Validation: Test Cases

The PCGO model has been implemented in the Radiative-Hydrodynamic code CHIC of the CELIA, using the pre-existing RT model for central ray computation. An efficient projection algorithm has been implemented to interpolate quantities from the PCGO predictions to the hydrodynamical mesh. Preliminary tests suggest that computation time is similar between RT runs using 2500 rays and PCGO runs using 1 thick ray. This model has been validated for the theoretical test cases of Gaussian beam diffraction in free-space and in constant density media. Furthermore, the numerical integration of the Riccati equation for B (eq. (10)) has been validated for the case of a linear density ramp. Similarly, solutions for wave propagation in a waveguide have been verified against analytical predictions.

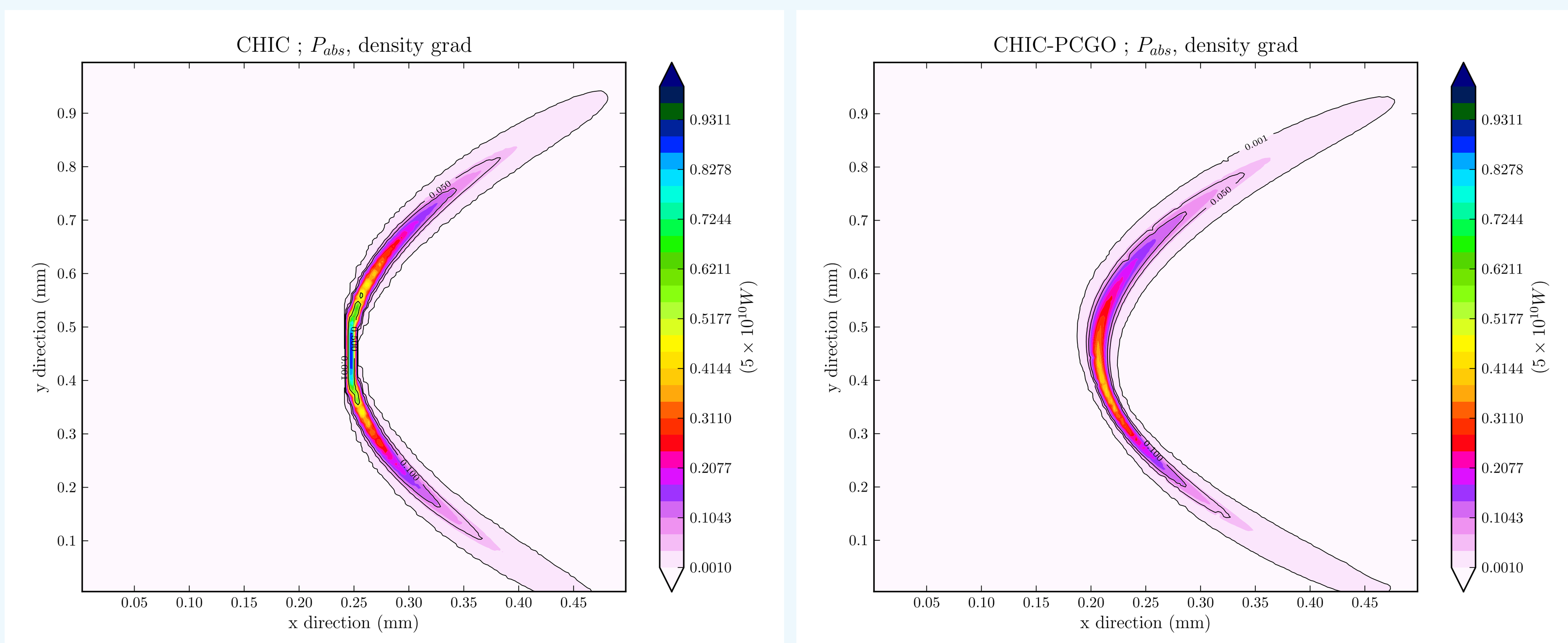


[Left] Normalized radius of a thick ray incident at $\theta_0 = 30°$ on a linear density ramp. Results from PCGO in CHIC are shown as a solid line and numerical integration of the Riccati equation using the analytical form for α [3] is shown as crosses. [Right] Normalized radius of a thick ray propagating along the axis of a waveguide for different values of the characteristic length $L_\perp$ of the density gradient. Theoretical solutions are shown as circles and results from PCGO in CHIC as solid lines. These numerical integrations are found to be in agreement with the theoretical solution for w.

Absorption, comparison with standard RT



[Top-left](RT) and [top-right](PCGO) show inverse Bremsstrahlung power deposited by a $w_0 = 21\mu$m Gaussian beam at $\lambda_0 = 1.05\mu$m in a $n_e/n_c = 0.3$ plasma. The Rayleigh length is large, so that the ray has an almost constant thickness over the 900μm of propagation. [Bottom-left](RT) and [bottom-right](PCGO) show power deposited by a narrow Gaussian beam with $w_0 = 5.25\mu$m in a $n_e/n_c = 0.3$ plasma. The Rayleigh range is 16 times smaller so that diffraction effects are more important. Repartition of deposited power is very different and highlights one of the short-comings of the RT technique. Although such narrow beams are unlikely to be found in ICF, these thicknesses are comparable to speckle sizes.



Power deposited in a linear density ramp. $n_e/n_c = 1$ at $x = 0$ mm and $n_e/n_c = 0$ at $x = 0.5$mm. The introduction of beam thickness and intensity profiles in PCGO results in a different spatial location of beam maximum of absorption and different value of the maximum absorbed power (although the total absorbed power is the same).

References

[1] B. T. Kaiser. Laser ray tracing and power deposition on an unstructured three-dimensional grid. *Physical Review E*, 61, 2000.

[2] Y. A. Kravtsov and P. Berczynski. Gaussian beams in inhomogeneous media: a review. *Studia Geophysica et Geodaetica*, 51, 2007.

[3] Y. A. Kravtsov and N. Y. Zhu. *Theory of Diffraction, Heuristic Approaches.* Alpha Science Series on Wave Phenomena. Alpha Science International LTD., Oxford, U.K., 2010.

[4] C. E. Max. Strong self-focusing due to the ponderomotive force in plasmas. *Physics of Fluids*, 19, 1997.

Conclusion and perspectives

A thick ray model has been formulated and implemented in the hydrocode CHIC, which allows to evaluate the beam electric field, radius of curvature, thickness and intensity at all points in the plasma. This model consistently takes into account absorption by inverse Bremsstrahlung and has been validated against several comprehensive test cases. Comparisons with standard Ray-Tracing has highlighted the advantages of a laser description which takes into account diffraction and refraction on index gradients. This model opens the way for taking into account nonlinear laser plasma interactions in a large scale code. For instance, the ponderomotive force has already been implemented and is being validated against theoretical results [4]. The next step planned with PCGO is the inline modeling of Cross Beam Energy Transfer.