LECTURE # 3

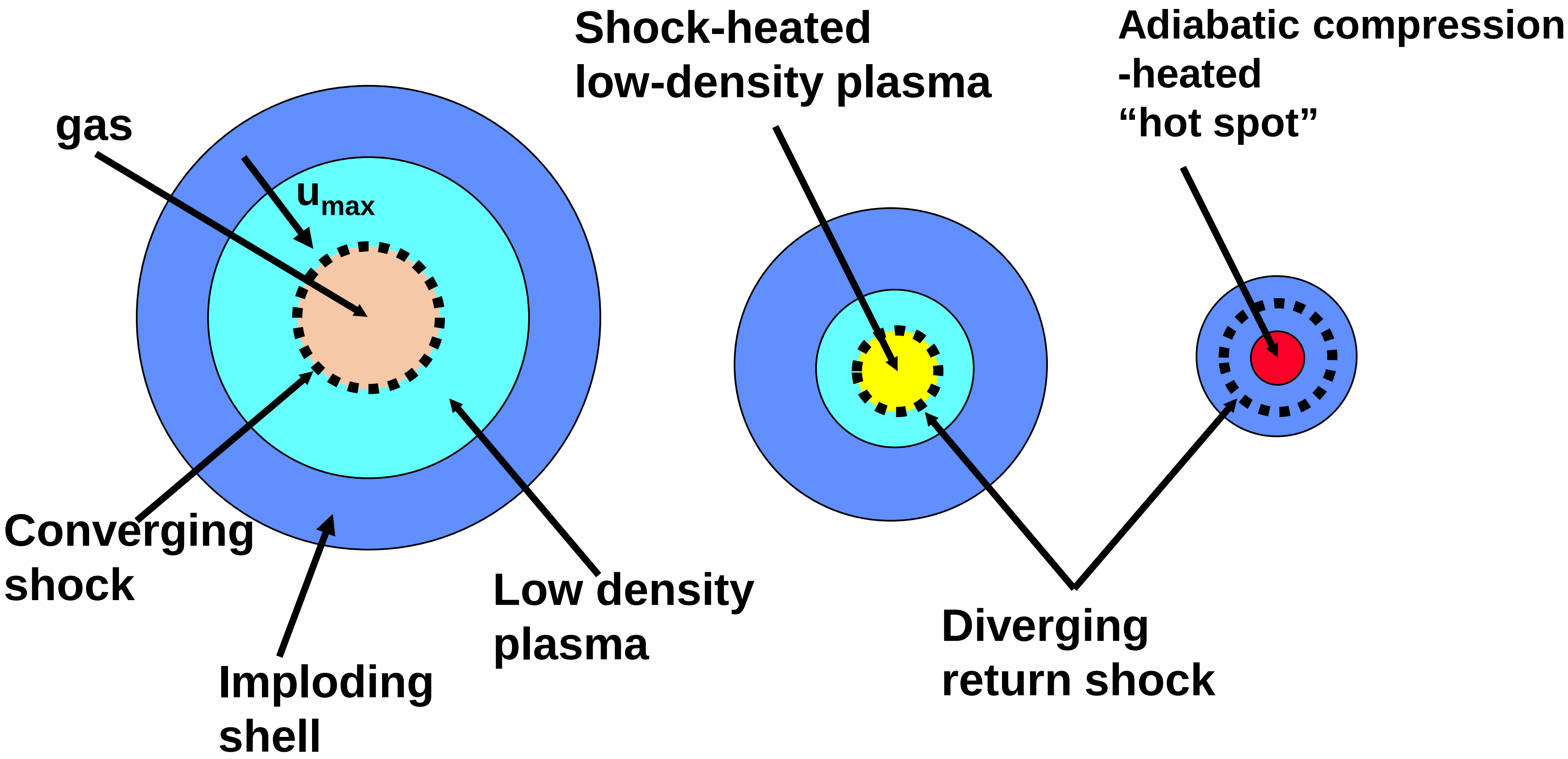
HOT SPOT DYNAMICS AND HYDRODYNAMIC INSTABILITIES

R. Betti

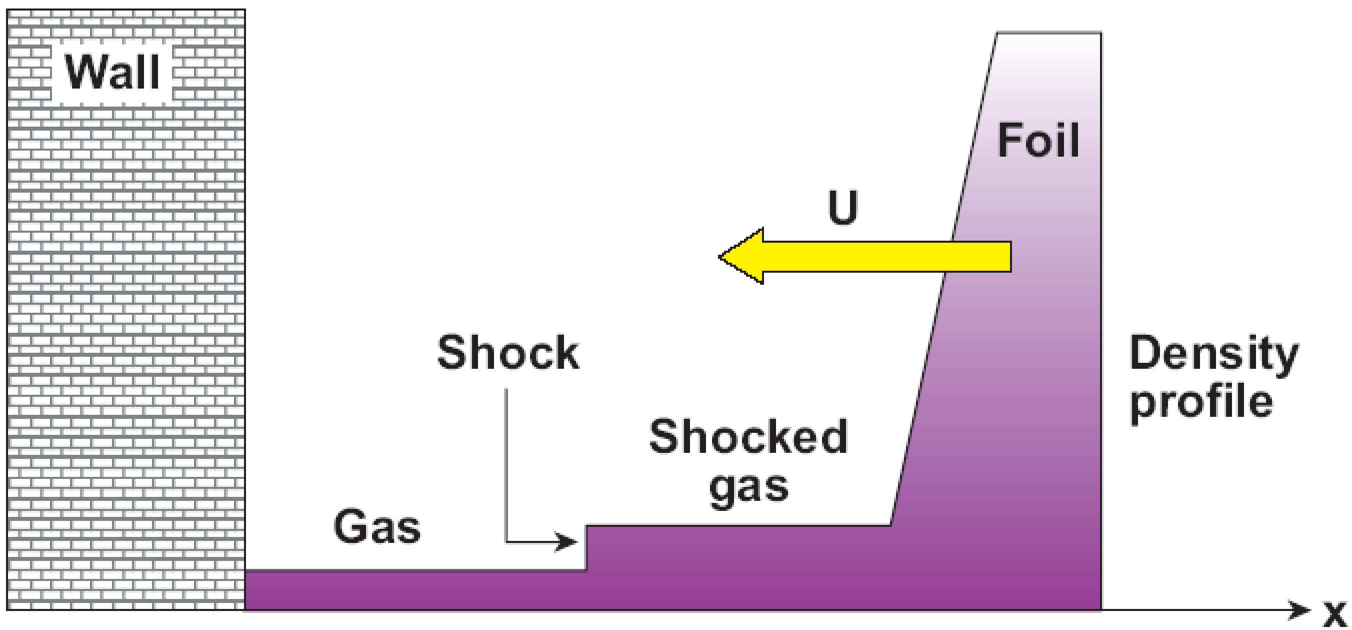
Fusion Science Center for Extreme States of Matter and Laboratory for Laser Energetics, University of Rochester

HEDP Summer School, July 14-20 2013, Ohio State University

Hot spot dynamics in 1D spherical implosions



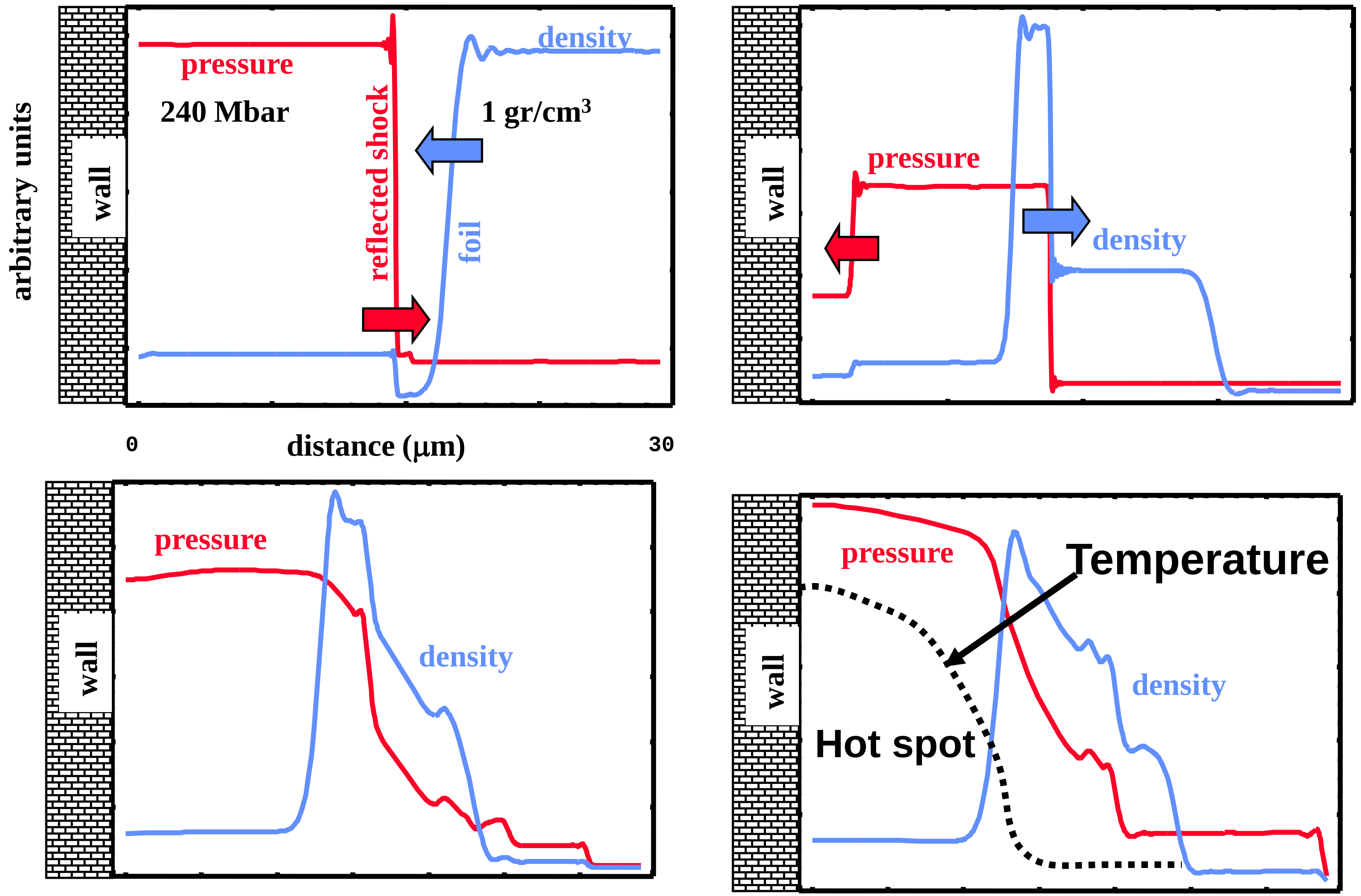
The decelerating slab problem provides the basic understanding of the deceleration phase and hot spot formation



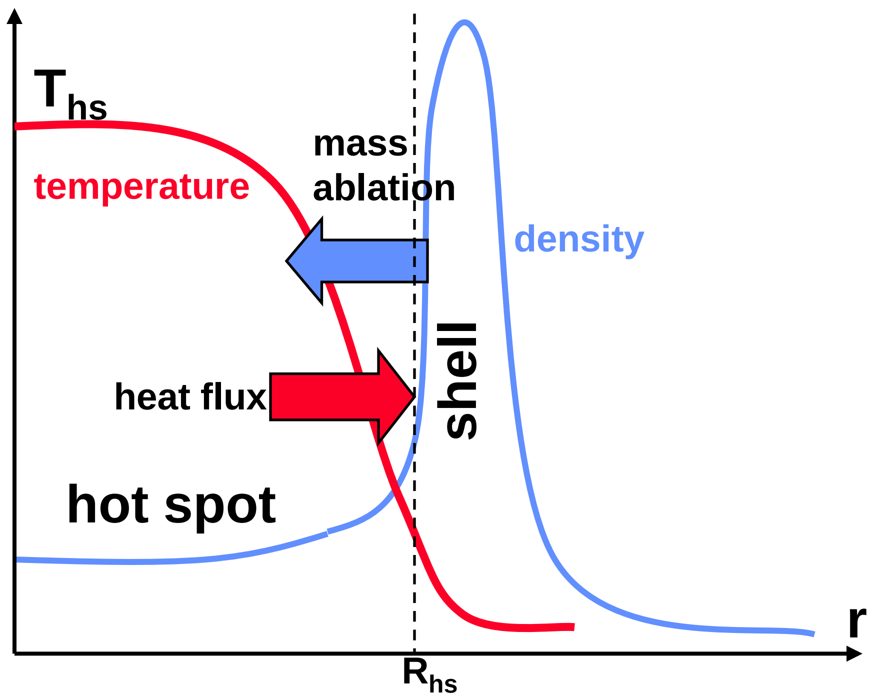
- The shock reflected from the wall slows down the foil, which in turn compresses the gas and decelerates.
- The 1-D problem can be solved analytically leading to a clear understanding of the relevant physics issues.

The low density gas is heated to form a hot spot

Evolution of pressure (red) and density (blue).



The heat flux leaving the hot spot is deposited onto the shell surface causing mass ablation from the shell into the hot spot. The hot spot mass increases in time.



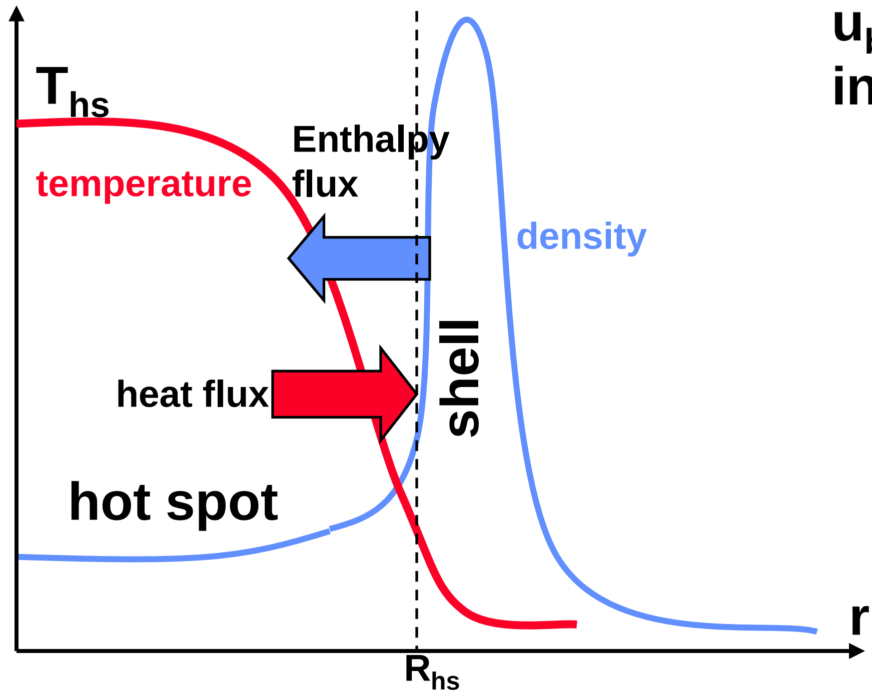
$$q_{heat} = -\kappa(T)\nabla T$$

$$\kappa(T) \approx \kappa_0 T^{5/2}$$

The heat leaving the hot spot cannot penetrate the shell because the shell is cold and its thermal conductivity is low,

$$\kappa_{shell} << \kappa_{hot\ spot}$$

The heat is deposited on the shell inner surface causing mass ablation off the shell



u_b = blow-off velocity into hot spot

$$\frac{5}{2} p u_b = \left[-\kappa_0 T^{5/2} \frac{dT}{dR_{hs}} \right]_{R_{hs}^-}$$

Enthalpy flux — **Heat flux**

- **Hot spot temperature profile →** $T_{hs} = T_0 \left(1 - \frac{r^2}{R_{hs}^2} \right)^{2/5}$
- **Use ideal gas EOS:** $p u_b = 2 \rho_{R_{hs}} T_{R_{hs}} u_b / M_i = 2 \dot{m} T_{R_{hs}} / m_i$
- **Ablation rate into hot spot:** $\dot{m} = \rho_{R_{hs}} u_b$

substitute — **find**

$$\dot{m} = 0.2 \frac{m_i \kappa_0 T_0^{5/2}}{R_{hs}}$$

Hot spot density

Hot spot volume ~ R_{hs}^3

Use EOS $\rho = m_i p / 2T$

$$M_{hs} = \rho_{hs} V_{hs} = \frac{m_i}{2} \frac{pV_{hs}}{T_{hs}}$$

Mass conservation →

$$\frac{dM_{hs}}{dt} = 4\pi R_{hs}^2 \dot{m} \;\Rightarrow\; m_i \frac{pV_{hs}}{tT} \sim R_{hs}^2 \frac{m_i \kappa_0 T^{5/2}}{R_{hs}}$$

Hot spot compression time →

$$t \sim \frac{R_{hs}}{u_{\max}}$$

$$pV_{hs} \sim M_{shell} u_{\max}^2$$

← **Energy Conservation (hot spot internal energy comes from shell kinetic energy)**

$$M_{shell} \sim \rho R^3$$

Hot spot → temperature

$$T \sim \left(\frac{M_{shell} u_{\max}^3}{\kappa_0 R_{hs}^2} \right)^{2/7} \sim \frac{1}{\kappa_0^{2/7}} \left[(\rho R)_{shell}^{stag} \right]^{2/7} u_{\max}^{6/7}$$

Shell areal density

- **Use the areal density scaling found in previous lecture**

$$(\rho R)_{stag}^{shell} \sim \frac{E_L^{1/3} u_{\max}^{2/3} I^{4/45}}{\alpha^{4/5}}$$

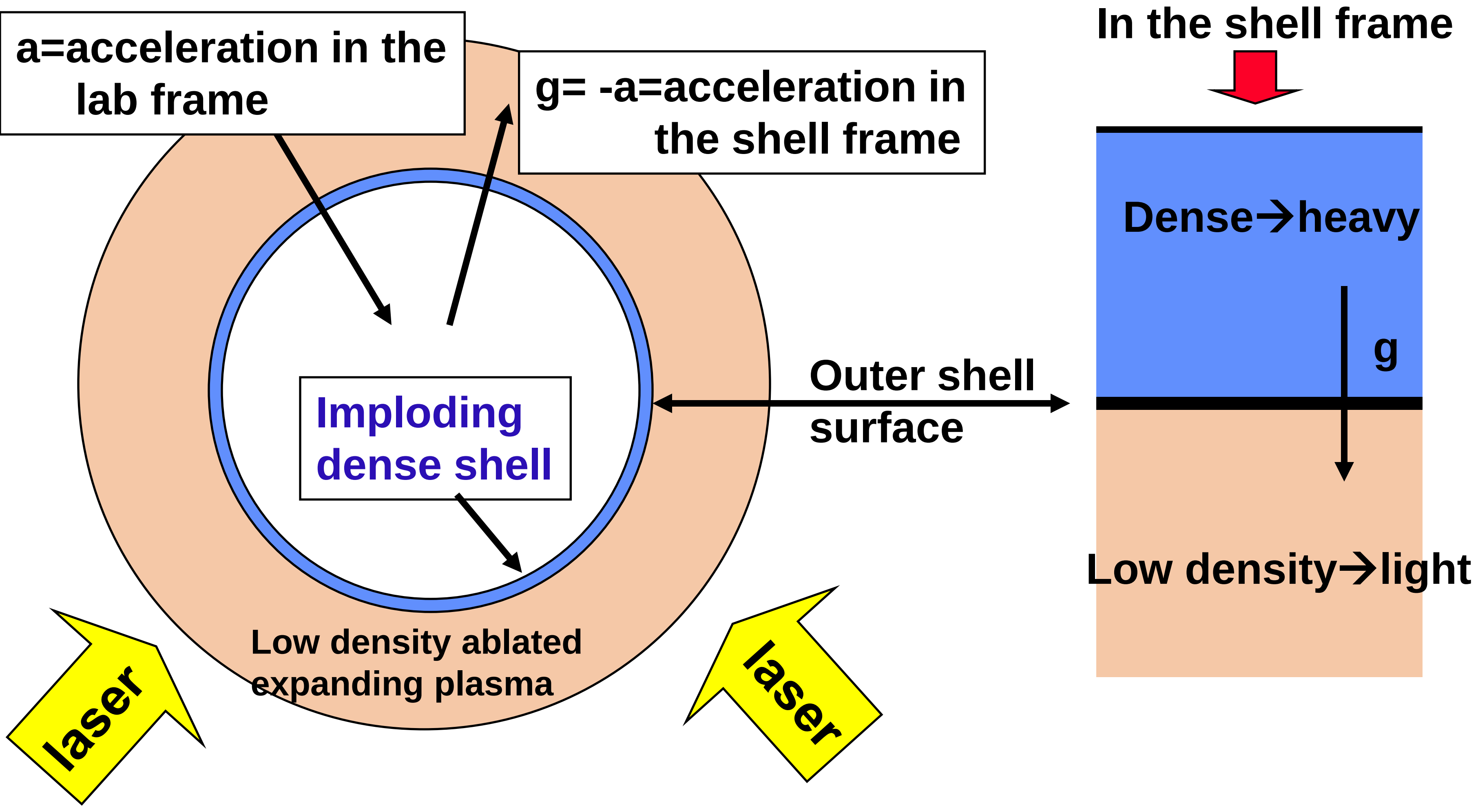
- **Find the temperature scaling. The hot spot temperature mainly depends on the implosion velocity.**

$$T_{hot-spot} \sim \frac{E_L^{0.1} I^{0.03}}{\alpha^{0.2}} u_{\max}^{1.1}$$

- **Simulation results (without α-heating) confirm the theory**

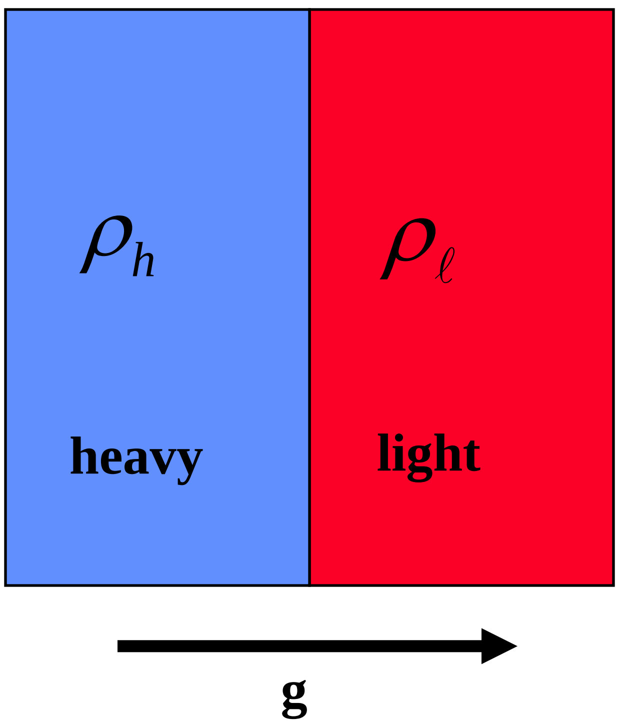
$$\langle T_{hot-spot} \rangle (keV) \approx \frac{3}{\alpha^{0.15}} \left(\frac{u_{\max}(cm/s)}{3\times 10^7} \right)^{1.25} \left(\frac{E_L(kJ)}{100} \right)^{0.07}$$

HYDRODYNAMIC INSTABILITIES

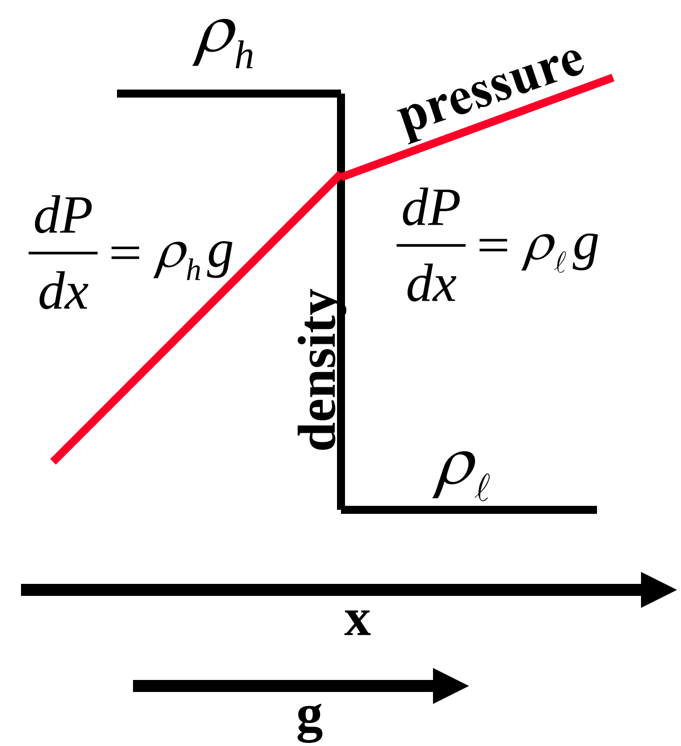


THE CLASSICAL RAYLEIGH-TAYLOR INSTABILITY
of
A HEAVY FLUID SUPPORTED BY A LIGHTER FLUID

EQUILIBRIUM CONDITIONS

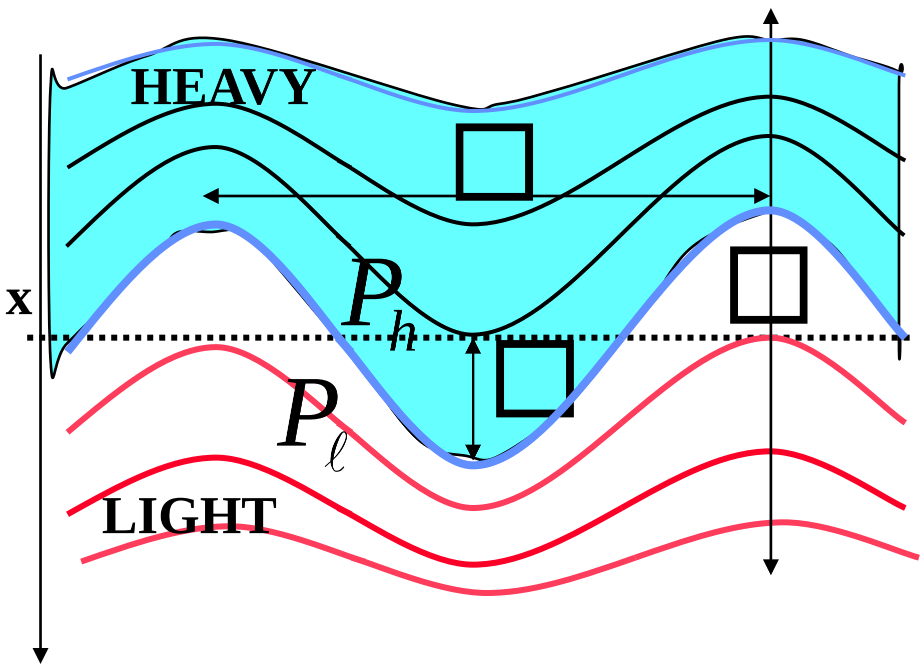


Pressure gradient is opposite to density gradient



Acceleration Phase

The classical R-T is just Newton's law at work: F=ma!



$$F = S(P_h - P_\ell) = ma = \rho_h \lambda S \ddot{\eta}$$

$$\frac{dP_0}{dx} = \rho_0 g = \begin{cases} \rho_h g & heavy \\ \rho_\ell g & light \end{cases}$$

$$P_\ell = P_0 + \left[\frac{dP_0}{dx}\right]_\ell \eta = P_0 + \rho_\ell g \eta$$

$$P_h = P_0 + \left[\frac{dP_0}{dx}\right]_h \eta = P_0 + \rho_h g \eta$$

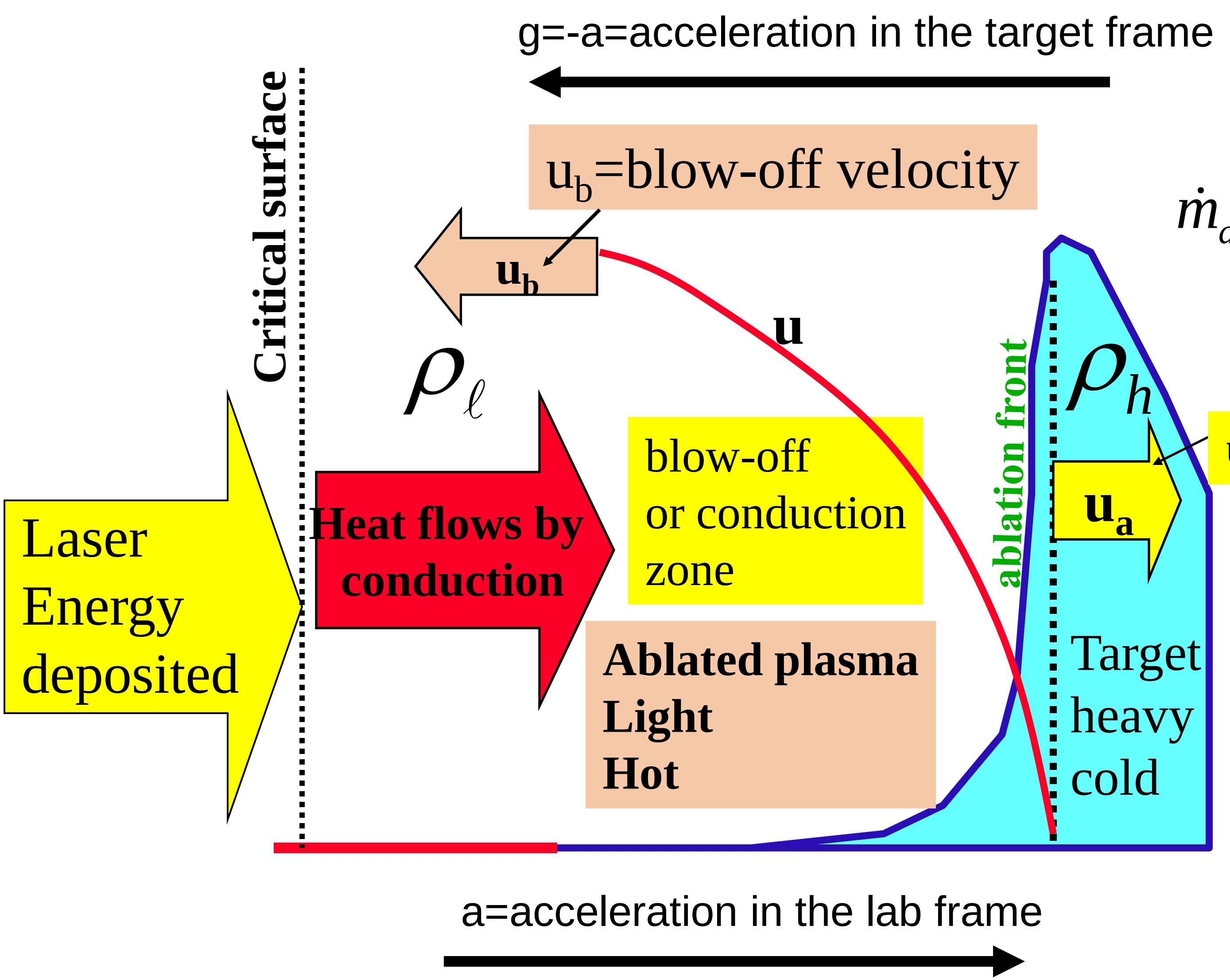
$$k \sim 1/\lambda$$

$$F = ma \text{--}> S\rho_h g\eta = \rho_h \lambda S \ddot{\eta} \text{--}> \ddot{\eta} = kg\eta \text{--}> \eta \sim e^{\gamma t} \text{--}> \gamma = \sqrt{kg}$$

In the laser irradiated targets the heat/ablation front penetrates at the ablation velocity

$$u_a = \frac{\dot{m}_a}{\rho_h}$$

$$\dot{m}_a = \rho_h u_a = \rho_\ell u_b$$



The ablation velocity is the speed at which the ablation front penetrates into the target. It can be calculated from the 1D theory results

$$\dot{m}_a = 3.3 \bullet 10^5 (I_{15} / \lambda_L^4)^{1/3} \, g / cm^2 s$$

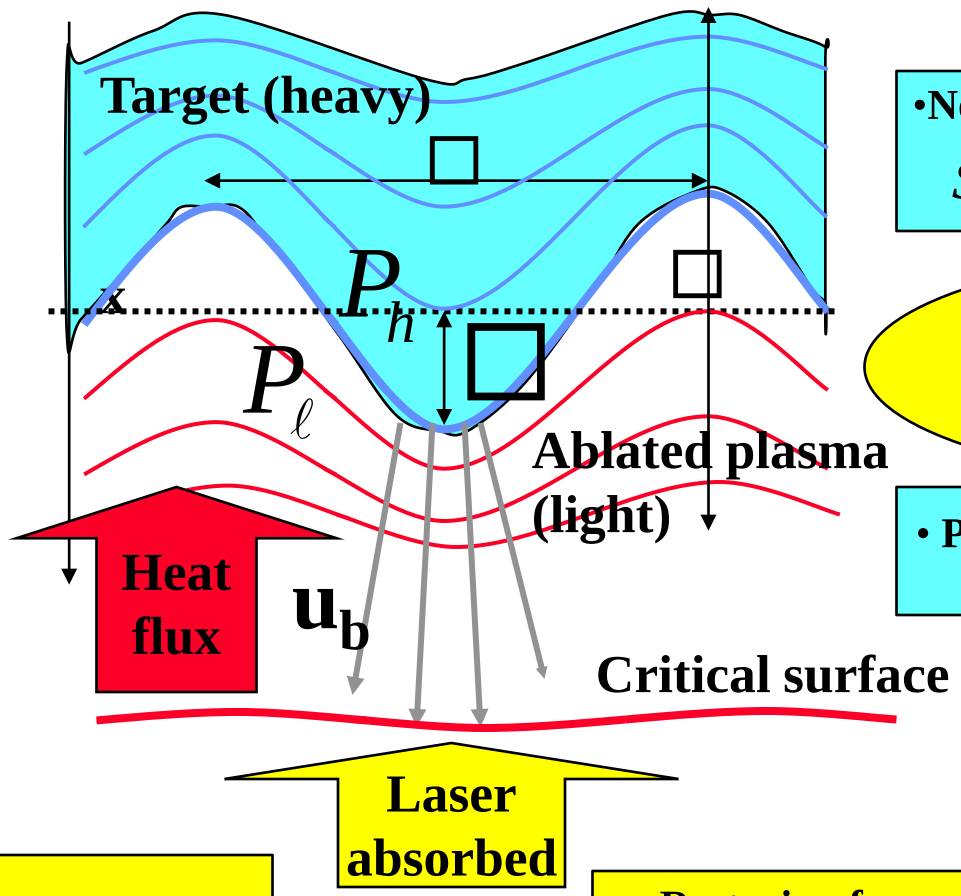
$$\dot{m}_a = \rho_{shell} u_a \qquad\qquad u_a = \dot{m}_a / \rho_{shell}$$

$$\rho_{shell}(g/cc) = \left(\frac{P(Mbar)}{2\alpha} \right)^{3/5}$$

$$P_a = 83 \left(\frac{I_{15}}{\lambda_{L(\mu m)} / 0.35} \right)^{2/3} Mbar$$

$$u_a = 1.1 \bullet 10^5 \frac{\alpha^{0.6}}{I_{15}^{0.067}} \left(\frac{0.35}{\lambda_L} \right)^{0.93} cm/s$$

The ABLATIVE R-T is just Newton's law at work again but with a restoring force: the dynamic pressure.



•Newton's law

$$S[P_h - (P_\ell + \rho_\ell u_b^2)] = \rho_h \lambda S \ddot{\eta}$$

•Energy balance

$$\frac{5}{2} p u_b = q_{heat} \Rightarrow \tilde{u}_b = u_b \eta / \lambda$$

Need to show

• Perturbed dynamic pressure

$$\underbrace{\rho_\ell u_b}_{\text{Ablation rate}} \tilde{u}_b = \dot{m} u_b \eta / \lambda$$

Ablation rate

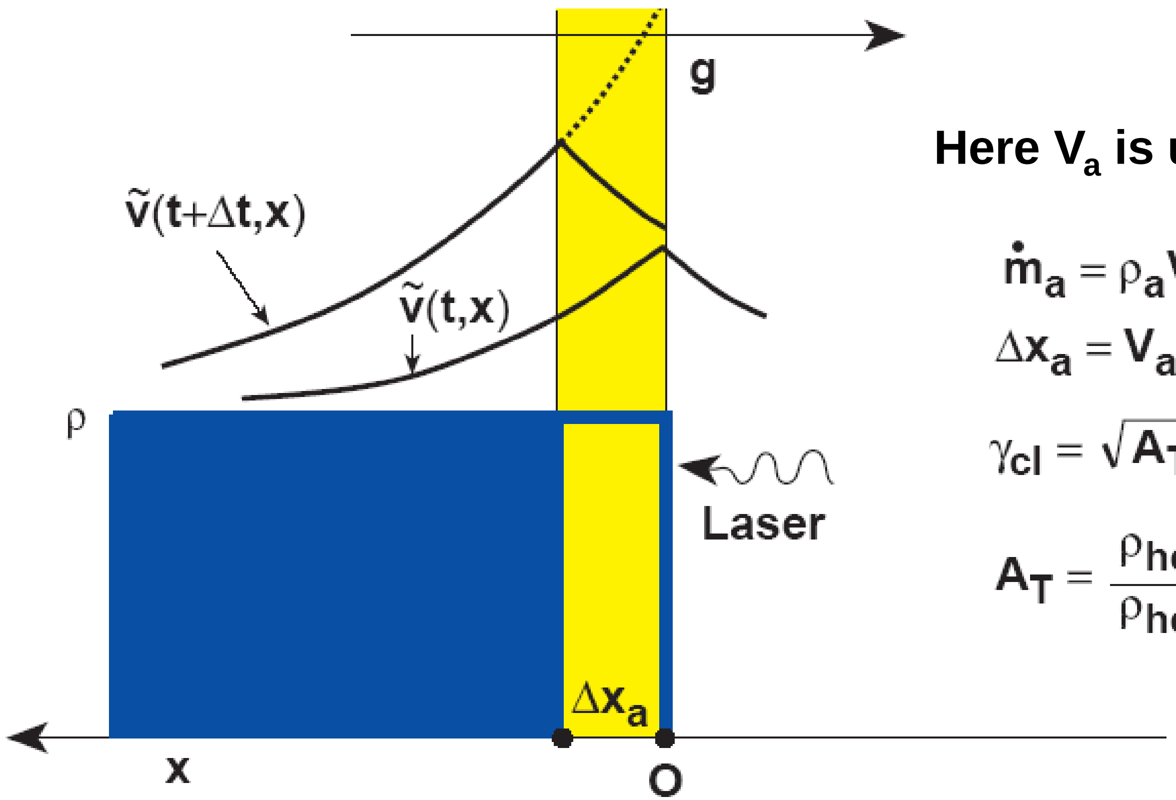
Restoring force

stabilizing

• Growth rate:

$$S(\rho_h g - k \dot{m} u_b)\eta = \rho_h \lambda S \ddot{\eta} -> \eta \sim e^{\gamma t} -> \gamma = \sqrt{kg - k^2 \frac{\dot{m}}{\rho_h} u_b}$$

Another stabilizing effect is the physical removal of the perturbation through ablation



Here V_a is u_a=ablation velocity

$$\dot{m}_a = \rho_a V_a$$

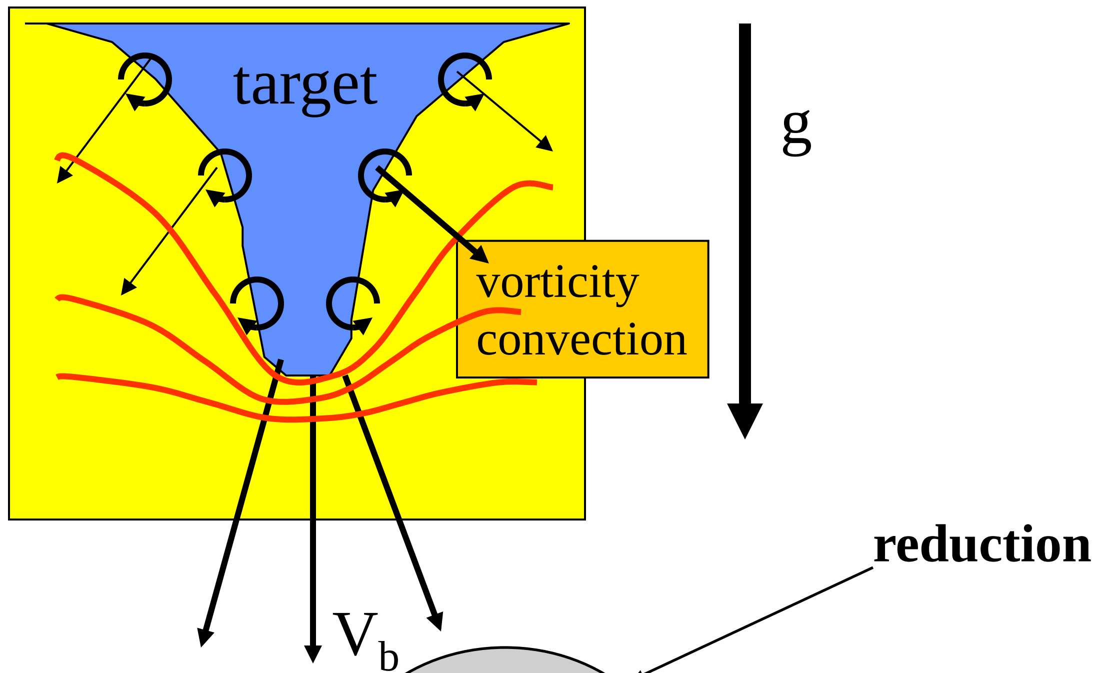
$$\Delta x_a = V_a \Delta t$$

$$\gamma_{cl} = \sqrt{A_T k g}$$

$$A_T = \frac{\rho_{heavy} - \rho_{light}}{\rho_{heavy} + \rho_{light}}$$

- Classical: $\tilde{v}(t,x) \sim e^{-kx}$
- Front frame (x'): $x = x' + V_a t$
- In the front frame: $\tilde{v}(t,x') \sim e^{(\gamma_c - kV_a)t - kx'} \rightarrow \gamma = \gamma_{cl} - kV_a$

Another stabilizing effect is the ablation-driven convection of the vorticity off the ablation front



$$\gamma = \gamma_{cl} - k u_a$$

A cutoff in the unstable spectrum limits the number of unstable modes

UR LLE

- **The full Ablative-RT growth rate includes all these effects:**

$$\gamma = \sqrt{Akg - k^2 \frac{\dot{m}}{\rho_h} u_b + 4k^2 u_a^2} - 2ku_a$$

Mass removal + vorticity convection

dynamic pressure

Both of these effects cannot damp a perturbation

- **Atwood number:**

$$A = \frac{\rho_{heavy} - \rho_{light}}{\rho_{heavy} + \rho_{light}}$$

- **The cutoff wave number depends only on the dynamic pressure:**

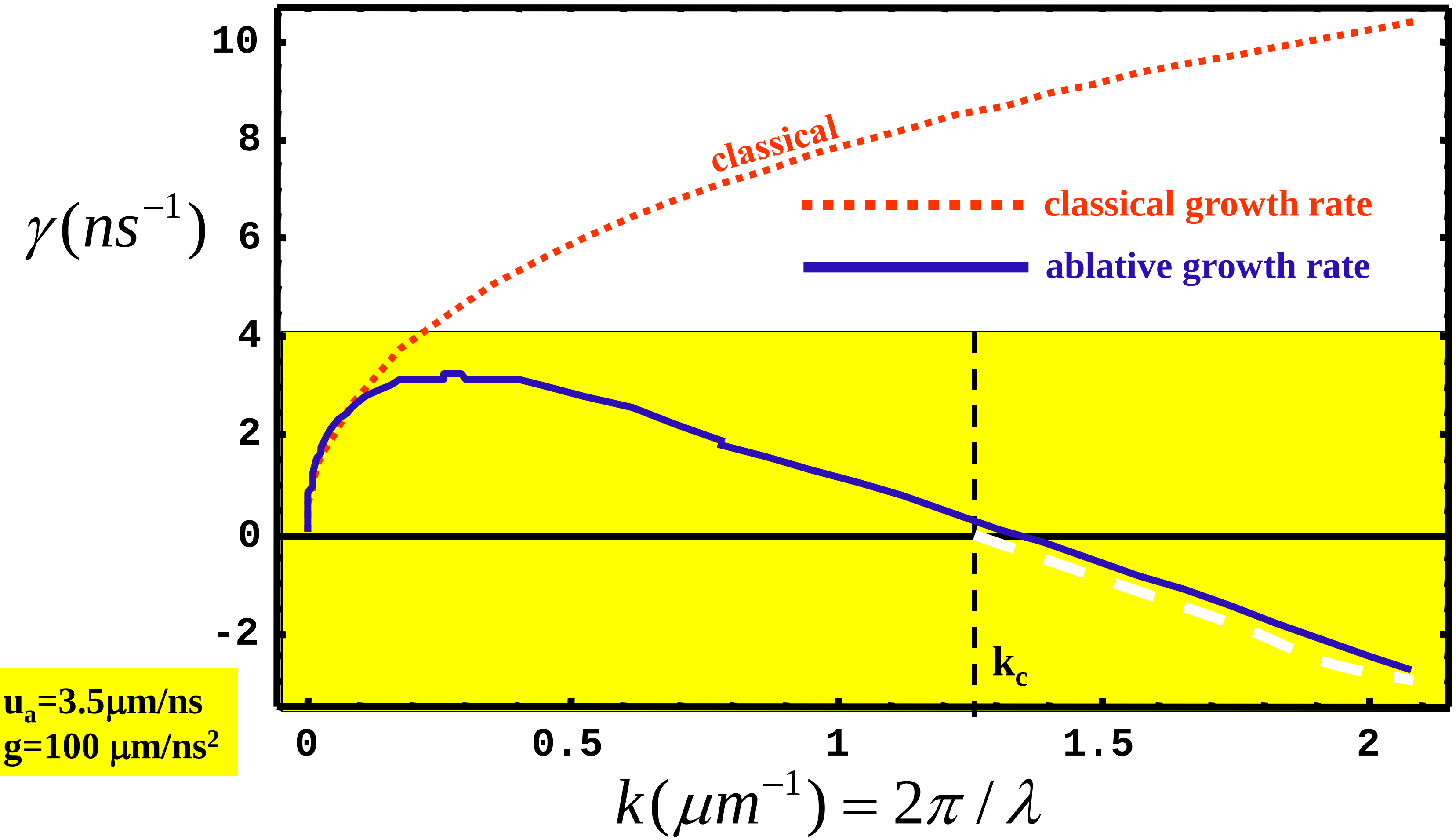
$$kg = k^2 \frac{\dot{m}}{\rho_h} u_b \; --> \; k_{cutoff} = \frac{\rho_h g}{\dot{m} u_b}$$

$$u_b = u_a \frac{\rho_h}{\rho_\ell}$$

- **Numerical fit (Takabe's formula):**

$$\gamma \approx 0.9\sqrt{kg} - 3ku_a$$

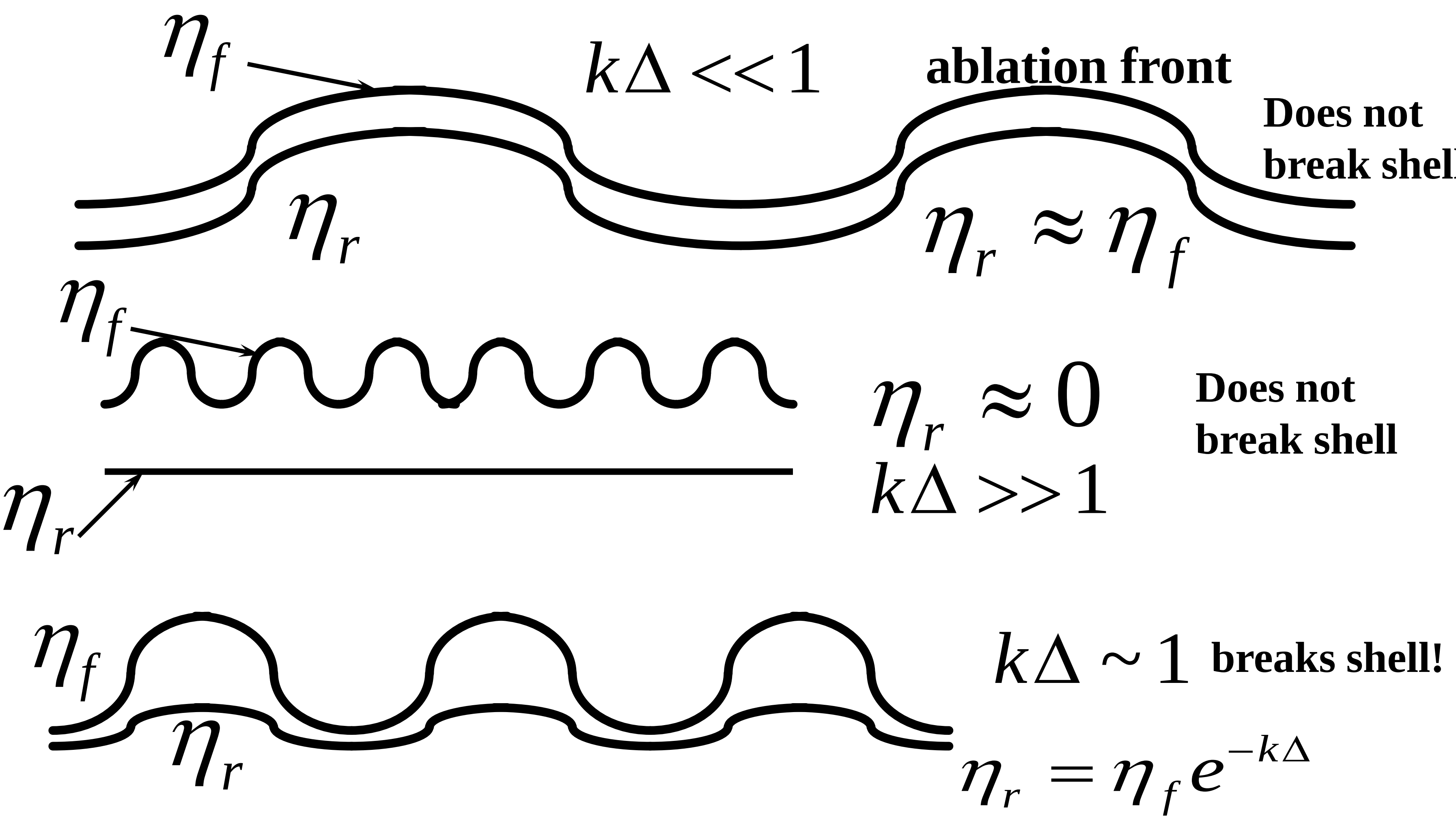
The ablative growth is significantly less than the classical value. Modes with k> k_c are stable



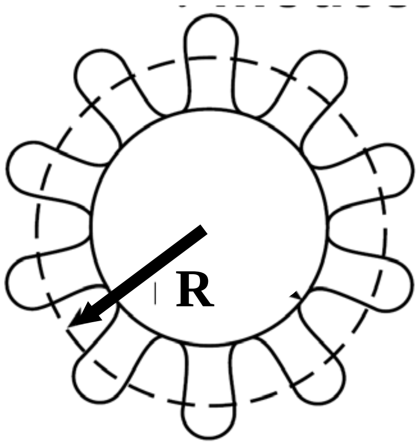
u_a=3.5µm/ns
g=100 µm/ns^2

Only modes with kΔ~1 break the targets because the distortion inside the target decays in space $\eta(x) = \eta_f e^{-kx}$

- Rear surface distortion → $\eta_r = \eta_f e^{-k\Delta}$



Most dangerous modes have mode number equal to the In-Flight-Aspect-Ratio IFAR



- **Wave number in planar geometry** $k = 2\pi / \lambda$

- **Wavelength in spherical geometry:** $\lambda = 2\pi R / \ell$

- **Wave number in spherical geometry** $k = 2\pi / \lambda = \ell / R$

- **Most dangerous modes in spherical geometry**

$$k\Delta = \ell\Delta / R = \ell / IFAR = 1 \quad \rightarrow \quad \ell_{most-dangerous} = IFAR$$

- **Aspect ratio of the target studied in previous lecture IFAR~70**

- **Most dangerous modes of our target→** $\ell \sim 70$

How much does a perturbation grow during the acceleration phase due to the (linear) RT instability?

UR LLE

$$\eta(t) = \eta(0)e^{\gamma t} \qquad \gamma = 0.9\sqrt{kg} - 3ku_a$$

$$\gamma t = 0.9\sqrt{kgt^2} - 3ku_a t = 0.9\sqrt{(k\Delta)\frac{gt^2}{\Delta}} - 3(k\Delta)\frac{u_a t}{\Delta}$$

$$\text{dist. travelled} = \frac{gt^2}{\not{2}} \approx \frac{R_0}{\not{2}} \qquad t = R_0 / u_{\max}$$

Take kΔ=1, most dangerous modes

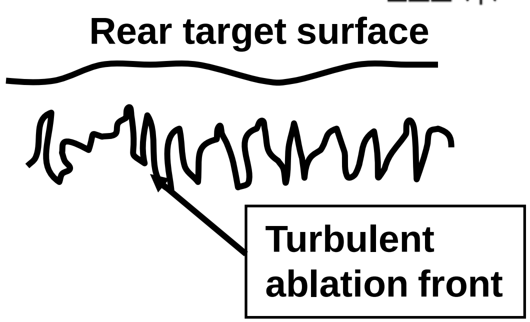
$$\gamma t = 0.9\sqrt{\frac{R_0}{\Delta}} - 3\frac{u_a}{u_{\max}}\frac{R_0}{\Delta} = 0.9\sqrt{IFAR} - 3\frac{u_a}{u_{\max}} IFAR$$

Use u_a=2.2•10^{5}cm/s, u_{max}=4.9•10^{7}cm/s, IFAR=70

$$\gamma t \approx 6.5 \quad \Rightarrow \quad \text{growth factor} = e^{\gamma t} = 665$$

What if the initial perturbations on the targets are so large that the RT become immediately nonlinear (→ multimode interaction) and a turbulent mixing front develop?

•Drop mode wavelengths as scale lengths



•Only scale length left is gt^2

•Mixing front of width h advances according to

$$h \sim \beta g t^2 = 2\beta Dist. \approx \beta R_0$$

• The figure of merit is the size of the mixing front to the target thickness

$$\frac{h}{\Delta} \approx \beta \frac{R_0}{\Delta} \approx \beta \bullet IFAR$$

•RT simulations gives $\beta \approx 0.05$

•Our target with IFAR=70 would be fully mixed → NO SHELL LEFT!

$$\frac{h}{\Delta} \approx 0.05 \bullet 70 = 3.5 > 1$$

MUST CONTROL THE SEEDS OF THE RT → MAKE SMOOTH TARGETS AND SMOOTH LASER BEAMS

Lot of work on hydrodynamic instabilities needs to be done

- Multimode, turbulent Rayleigh-Taylor instability is not well understood

- The effect of ablation on the nonlinear multimode evolution is not well understood

- The effect of the initial conditions on the turbulent front dynamics is not well understood

- This is important stuff for inertial fusion!