

LECTURE # 2

One dimensional implosion hydrodynamics

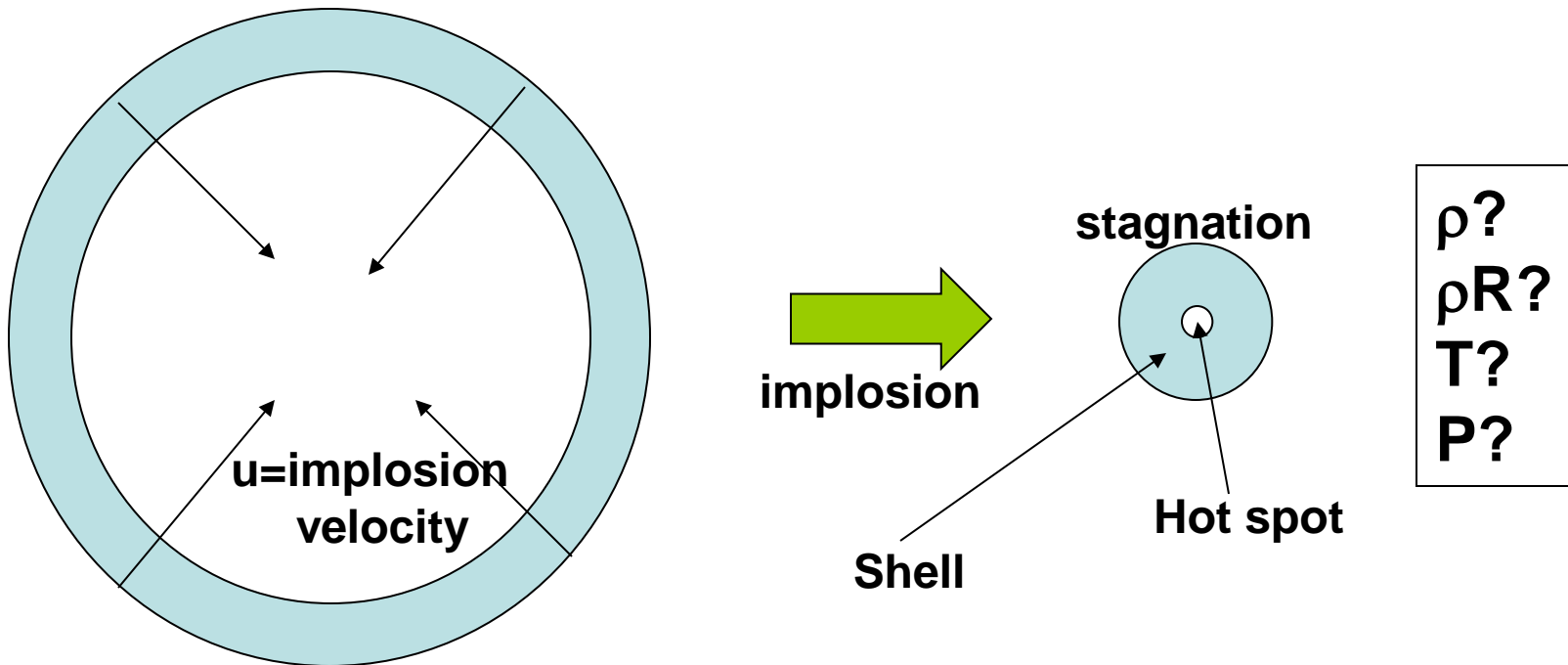
R. Betti

***Fusion Science Center for Extreme States of Matter
and Laboratory for Laser Energetics, University of Rochester***

HEDP Summer School, July 14-20, 2013, Ohio State University

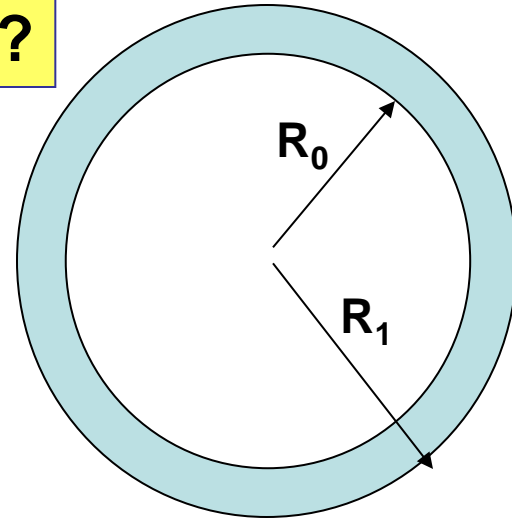
1D implosion hydrodynamic theory should answer the following question

- What are the stagnation values of the relevant hydrodynamic properties?

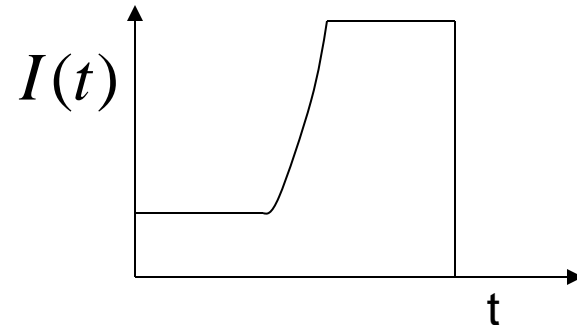


What variables can be controlled?

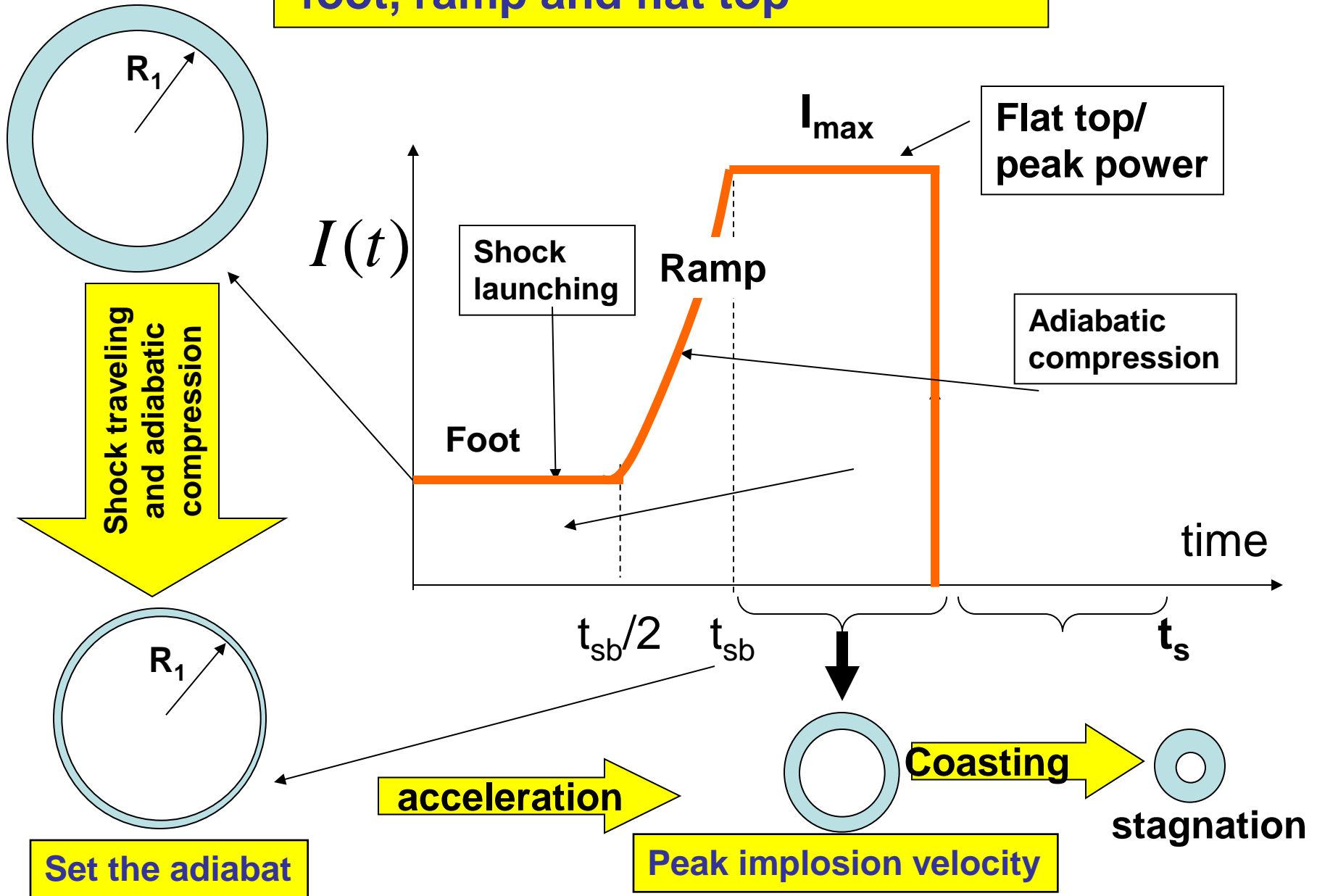
- (a) Shell inner radius R_0 at time $t=0$
- (b) Shell outer radius R_1 at time $t=0$
- (c) The total laser energy on target
- (d) Adiabatic (or Entropy) through shocks
- (e) Applied pressure $P(t)$ through the pulse shape $I(t)$ ← laser intensity



$$\alpha \sim \frac{P}{\rho^{5/3}} \quad P \sim I^{2/3}$$

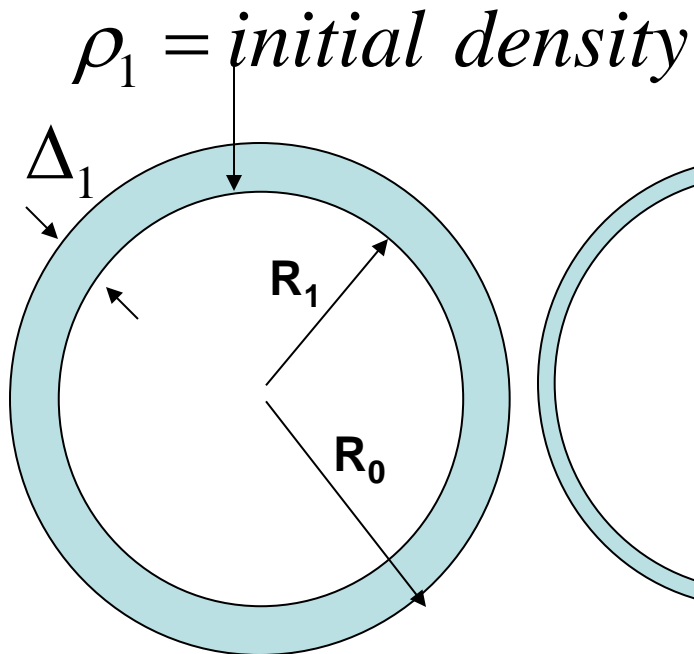
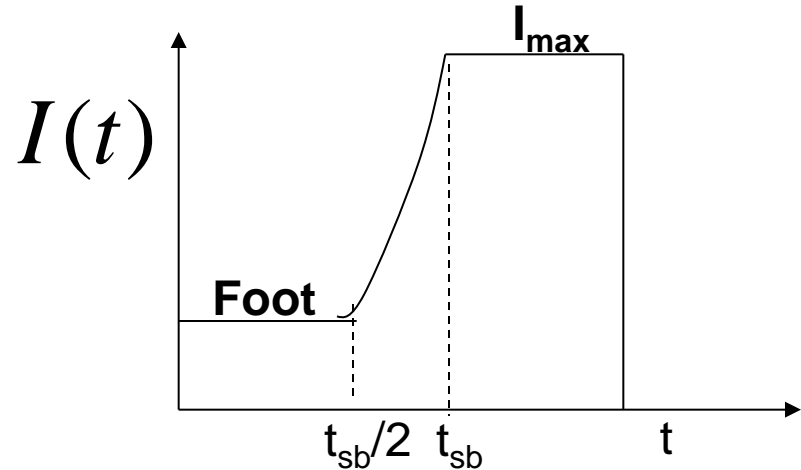


The laser pulse has three stages: foot, ramp and flat top



The adiabat is set by the shock launched by the foot of the laser pulse

$$\alpha \sim \frac{P_{foot}}{(4\rho_1)^{5/3}}$$



Shock break-out

$$\rho_{sb} \sim \left(\frac{P_{max}}{\alpha} \right)^{3/5} = 4\rho_1 \left(\frac{P_{max}}{P_{foot}} \right)^{3/5}$$

$$\Delta_{sb} \sim \frac{\Delta_1}{4} \left(\frac{P_{foot}}{P_{max}} \right)^{3/5} \sim \frac{\Delta_1}{4} \left(\frac{I_{foot}}{I_{max}} \right)^{2/5}$$

Density and thickness at shock break out

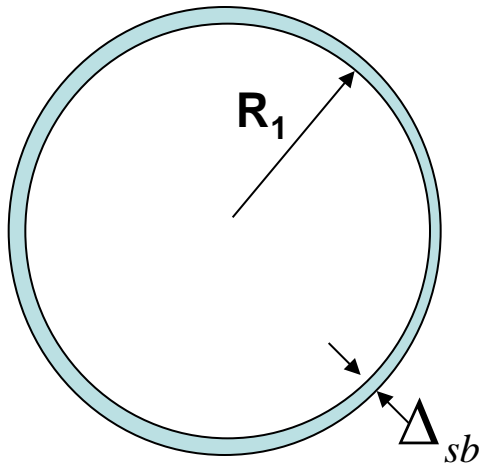
- Use $p \sim I^{2/3}$ to find:

Density \rightarrow
$$\rho_{sb} \sim 4\rho_1 \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}$$

Shell thickness \rightarrow
$$\Delta_{sb} \sim \frac{\Delta_1}{4} \left(\frac{I_{foot}}{I_{\max}} \right)^{2/5}$$

Shell radius \rightarrow
$$R \approx R_1$$

At shock break out the aspect ratio is maximum



$$A_1 = \frac{R_1}{\Delta_1} = \text{initial aspect ratio}$$

$$A_{sb} = IFAR = \frac{R_1}{\Delta_{sb}} = 4A_1 \left(\frac{I_{\max}}{I_{\text{foot}}} \right)^{2/5}$$

**IFAR = Aspect ratio at shock break-out =
Maximum In-Flight-Aspect-Ratio**

$$A_{sb} = A_{\max}$$

The IFAR scales with the Mach number

The shell kinetic energy is equal to the work done on the shell
(this relation should be improved by including the ablated mass)

$$Mu_{\max}^2 \sim \int_R^{R_1} pr^2 dr \sim p(R_1^3 - R^3)$$

Neglect
for $R \ll R_0$

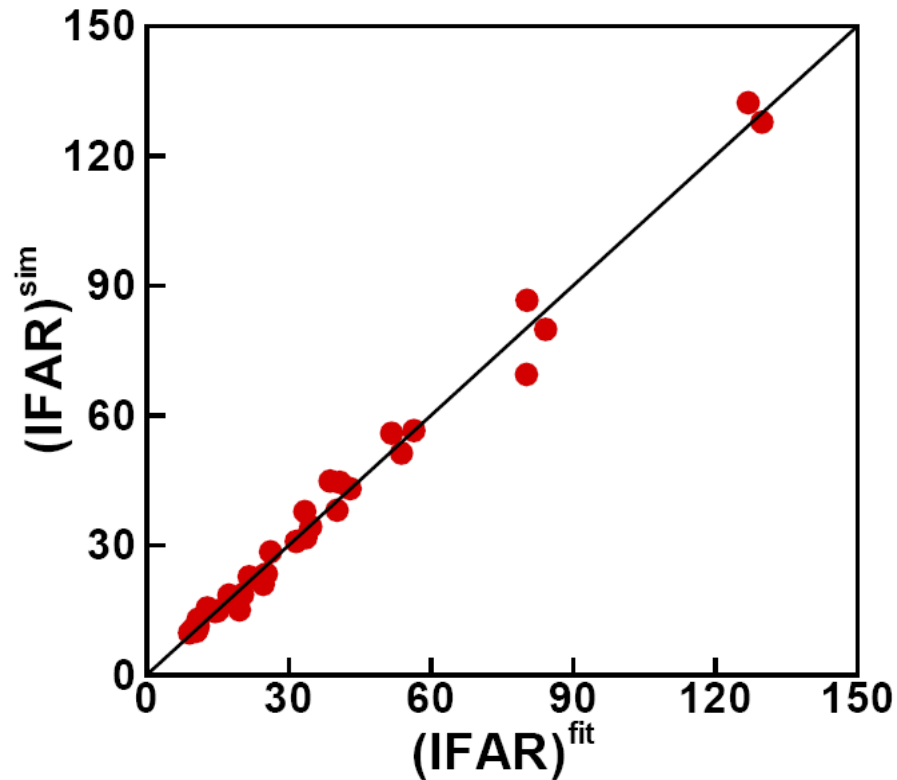
$$M \sim \rho_{sb} \Delta_{sb} R_1^2$$

$$IFAR = A_{sb} = \frac{R_1}{\Delta_{sb}} \sim \frac{u_{\max}^2}{p / \rho_{sb}} \sim Mach_{\max}^2$$

$$\rho \sim (p / \alpha)^{3/5} \quad p \sim I^{2/3}$$

$$IFAR = A_{sb} = \frac{R_1}{\Delta_{sb}} \sim \frac{u_{\max}^2}{\alpha^{3/5} I^{4/15}}$$

1D simulations confirm the IFAR scaling with the Mach number



$$IFAR_{\max}^{fit} = \frac{55 I_{15}^{-0.27}}{\alpha^{0.72}} \left(\frac{u_{\max} (cm/s)}{3 \bullet 10^7} \right)^{2.12}$$

The IFAR formula can be used to find the final implosion velocity.

$$u_{\max}^2 \sim IFAR \cdot \alpha^{3/5} I^{4/15}$$

Substituting the IFAR from shock breakout, we find the implosion velocity as a function of control variables

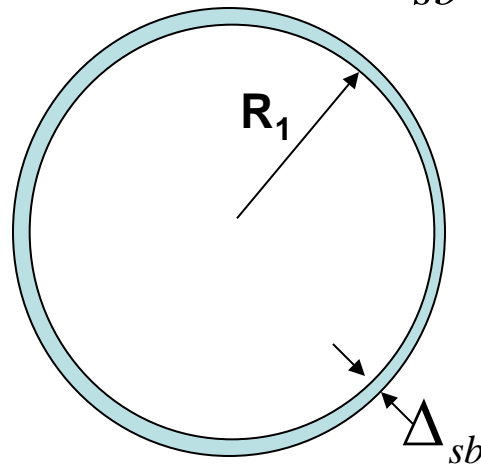
$$IFAR = 4A_1 \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}$$

$$u_{\max}^{cm/s} \approx 10^7 \sqrt{0.7 A_1 \cdot \alpha^{3/5} I_{15(\max)}^{4/15} \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}}$$

A simple implosion theory can be derived in the limit of infinite initial aspect ratio

- Start from a high aspect ratio shell at the beginning of the acceleration phase

$$IFAR = A_{sb} = \frac{R_1}{\Delta_{sb}} \gg 1$$



A first step in implosion hydrodynamic theory is the paper of Basko and Meyer-ter-vehn, Phys. Rev. Lett. 2003

**There are two important time scales:
the shell expansion time and the implosion time**

Shell expansion/contraction: $t_{ex} \sim \Delta / C_s$

Implosion time: $t_i \sim R / u_i$

$$\frac{t_i}{t_{ex}} \sim \frac{A}{Mach} \quad A = \frac{R}{\Delta} \quad Mach = \frac{u_i}{C_s}$$

In the acceleration phase $A \sim Mach^2$

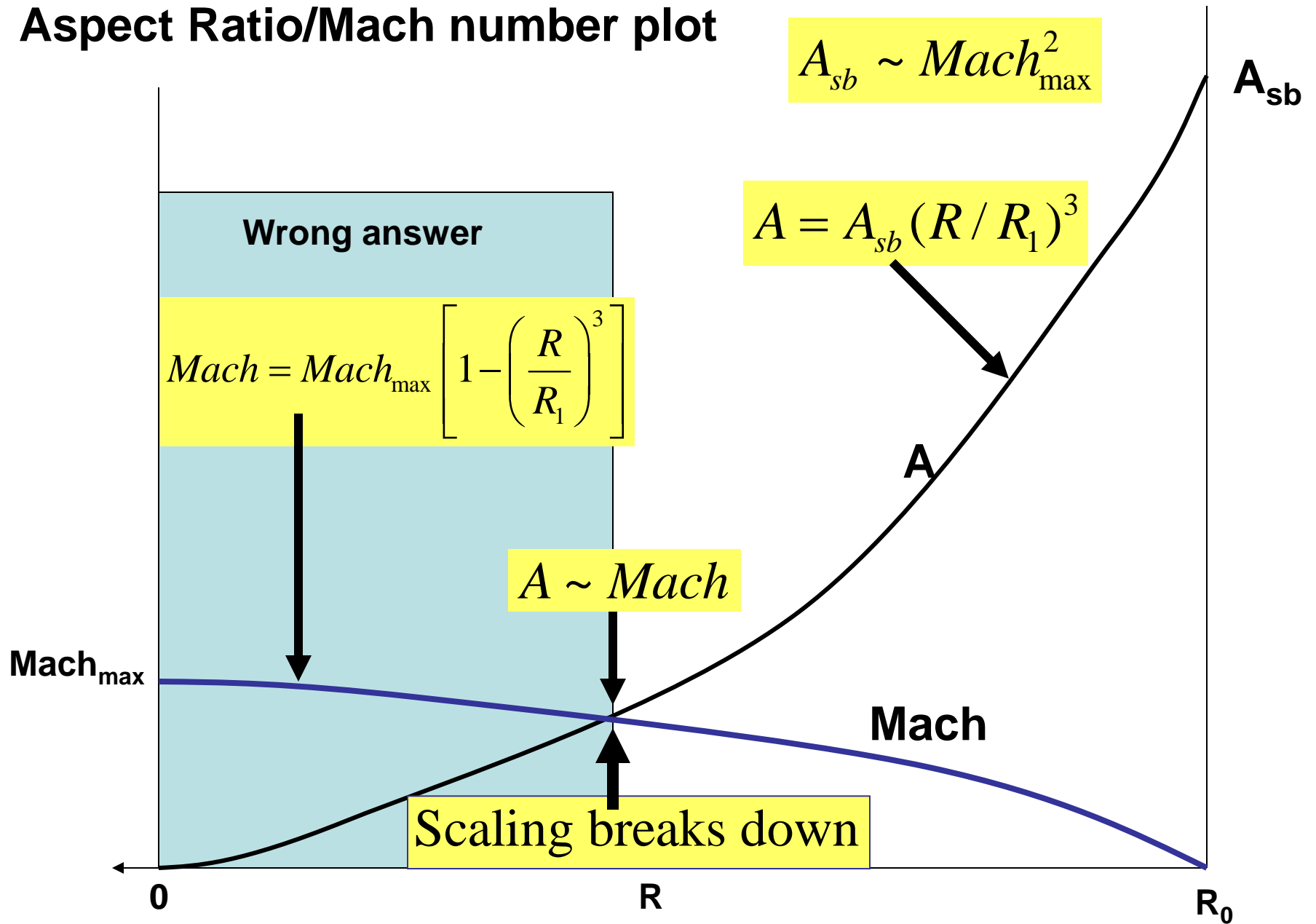
The density is constant

$$\frac{t_i}{t_{ex}} \sim \frac{A}{Mach} \sim Mach \sim \sqrt{A} \gg 1 \rightarrow \rho \approx const$$

$$\Delta \sim 1 / R^2$$

$$A = A_{sb} (R / R_1)^3$$

Aspect Ratio/Mach number plot



The acceleration phase ends when the implosion time is of the order of the compression time at $R_* \sim R_0 / A_{sb}^{1/6}$

$$A = A_{sb} (R / R_1)^3 \sim Mach_{\max}^2 (R / R_1)^3$$

When $(R / R_1) \sim (R_* / R_1) \sim 1 / A_{sb}^{1/6}$

$$A \sim A_* \sim A_{sb}^{1/2} \sim Mach_{\max}$$

The expansion time is of the same order of → the implosion time

$$\frac{t_i}{t_{ex}} \sim \frac{A_*}{Mach_*} \sim 1$$

For $R/R_1 < 1/A_{sb}^{1/6}$ the coasting phase begins and the shell thickness Δ is constant

$$(R / R_1) < (R_* / R_1) \sim 1 / A_{sb}^{1/6}$$

The implosion time
left (R_*/V_{\max})
is less than the
expansion time



$$\frac{t_i}{t_\Delta} < 1$$

The shell thickness
is constant



$$\Delta \sim \text{const} = \Delta_*$$

$$\Delta_* \sim \frac{\Delta_*}{R_*} R_* \sim \frac{R_*}{A_*} \sim \frac{R_*}{R_1} \frac{R_1}{A_*} \sim \frac{1}{A_{sb}^{1/6}} \frac{R_1}{A_{sb}^{1/2}} \sim \frac{R_1}{A_{sb}^{2/3}}$$

In the coasting phase the Mach number decreases like $R^{2/3}$ and the aspect ratio like R

$$A = \frac{R}{h_*} = A_* \frac{R}{R_*}$$

$$Mass \sim \rho R^2 \Delta_* \Rightarrow \rho \sim 1/R^2$$

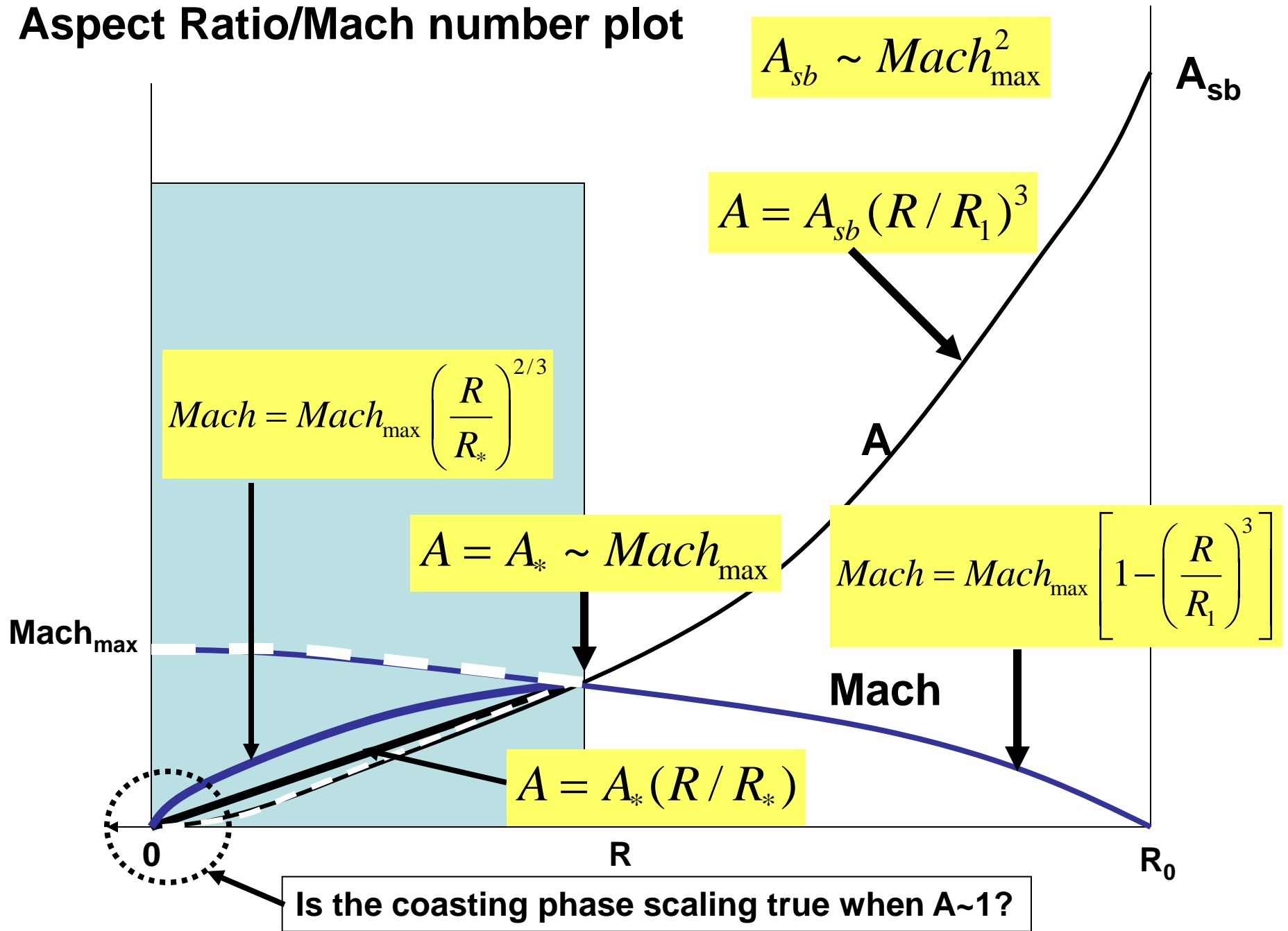
$$Mach \sim \frac{V_{\max}}{\sqrt{p/\rho}}$$

$$p \sim \alpha \rho^{5/3}$$

$$Mach \sim R^{2/3} \Rightarrow Mach = Mach_{\max} (R/R_*)^{2/3}$$

$$Mach \sim \sqrt{A_{sb}} (R/R_*)^{2/3}$$

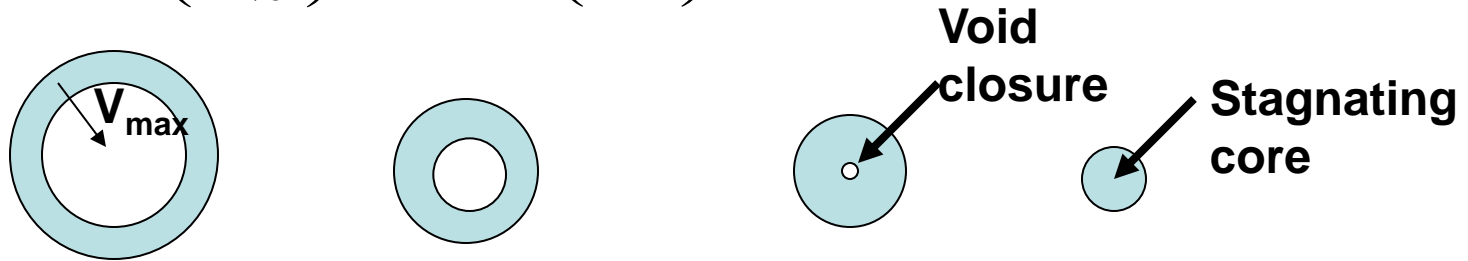
Aspect Ratio/Mach number plot



A scaling law can be derived for the density at stagnation

Before “void closure” (vc) the Aspect Ratio $A \sim 1 \rightarrow R_{vc} \sim \Delta_*$

$$\rho_{vc} = \rho_* \left(\frac{R_*}{R_{vc}} \right)^2 = \rho_{sb} \left(\frac{R_*}{\Delta_*} \right)^2 = \rho_{sb} A_*^2 = \rho_{sb} Mach_{\max}^2$$



The collapse of the shell generates a “return” shock that propagates from the center outward. The return shock compresses the core by a factor of 4 (not quite right)

$$\rho_{stag} \sim 4 \rho_{vc} \sim 4 \rho_{sb} Mach_{\max}^2 \sim 4 \rho_{sb} A_{sb}^2$$

Use $\frac{\rho_{sb}}{\rho_1} \sim 4 \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}$ \rightarrow $\rho_{stag} \sim 16 \rho_1 IFAR \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}$

The stagnation pressure scaling follows from energy conservation. The stagnation entropy scales as Mach^{2/3}

$$P_{stag} R_{stag}^3 \sim Mass \times u_{max}^2 \quad Mass \sim \rho_{stag} R_{stag}^3$$

$$P_{stag} \sim \rho_{stag} \times u_{max}^2 \sim P_{applied} \frac{\rho_{stag}}{\rho_{sb}} \frac{u_{max}^2}{P_{applied} / \rho_{sb}}$$

Stagnation pressure scaling

$$P_{stag} \sim P_{applied} Mach_{max}^4$$

Stagnation entropy scaling

$$\alpha_{stag} \sim \frac{P_{stag}}{\rho_{stag}^{5/3}} \sim \alpha Mach_{max}^{2/3}$$

Scaling of the areal density of the compressed core

$$\rho_{stag} R_{stag} \sim \rho_{stag}^{2/3} (\rho_{stag} R_{stag}^3)^{1/3} \sim (\rho_{sb} Mach^2)^{2/3} \left(\frac{E}{u^2} \right)^{1/3}$$

$$\rho_{sb} \sim \left(\frac{P_{applied}}{\alpha} \right)^{3/5} \quad Mach^2 \sim \frac{u^2}{\alpha^{3/5} I^{4/15}}$$

The areal density ρR scales as the laser energy E to the power 1/3.

$$\rho_{stag} R_{stag} \sim \frac{E^{1/3} u_{max}^{2/3} I^{4/45}}{\alpha^{4/5}}$$

Amplification of the areal density

$$\rho_{stag} R_{stag} \sim \rho_{stag}^{2/3} (\rho_{stag} R_{stag}^3)^{1/3} \sim \rho_{sb}^{2/3} Mach^{4/3} M^{1/3}$$

$$\rho_{stag} R_{stag} \sim \frac{\rho_{sb}^{2/3}}{\rho_1^{2/3}} Mach^{4/3} \rho_1^{2/3} (\rho_1 R_1^2 \Delta_1)^{1/3}$$

$$\rho_{stag} R_{stag} \sim (\rho_1 \Delta_1) Mach^{4/3} A_1^{2/3} \left(\frac{\rho_{sb}}{\rho_1} \right)^{2/3}$$

where

$$\frac{\rho_{sb}}{\rho_1} = 4 \left(\frac{I_{max}}{I_{foot}} \right)^{2/5}$$

IMPLOSION SCALING RECAP

$$A_{sb} = IFAR = 4A_1 \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}$$

$$u_{\max}^{cm/s} \approx 10^7 \sqrt{0.8A_1 \cdot \alpha^{3/5} I_{15(\max)}^{4/15} \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}}$$

$$\rho_{stag} \sim \rho_{sb} Mach_{\max}^2 \sim 16\rho_1 IFAR \left(\frac{I_{\max}}{I_{foot}} \right)^{2/5}$$

$$P_{stag} \sim P_{applied} Mach_{\max}^4 \sim P_{applied} IFAR^2$$

$$\alpha_{stag} \sim \alpha Mach^{2/3} \sim \alpha IFAR^{1/3}$$

$$(\rho R)_{stag} \sim (\rho_1 \Delta_1) IFAR^{2/3} A_1^{2/3} \left(\frac{I_{\max}}{I_{foot}} \right)^{4/9}$$

And the laser energy?

- The laser energy is included through the peak laser intensity and the shell outer radius

$$E_L = 4\pi R_0^2 I_{\max} t_{\text{imp}} \approx 4\pi R_0^2 I_{\max} \frac{R_0}{u_{\max}}$$

$$E_L \approx \frac{4\pi R_0^3 I_{\max}}{u_{\max}}$$

Homework problem

- Consider typical target shown in previous lecture
- $A_1 \approx 4$, $R_1 = 1350 \mu\text{m}$, $R_0 = 1700 \mu\text{m}$, $\Delta_1 = 350 \mu\text{m}$
- $\rho_1 = 0.25 \text{ g/cc}$
- $\rho_1 \Delta_1 = 0.009 \text{ g/cm}^2$
- $I_{\text{max}} = 10^{15} \text{ W/cm}^2$ leading to $P_{\text{max}} \sim 100 \text{ Mbar}$
- $I_{\text{foot}} = 2.2 \cdot 10^{13} \text{ W/cm}^2$ to set the shell on $\alpha = 3$

Use hydro-relations in previous slide to find:

$$IFAR \approx 70 \quad u_{\text{max}} \approx 4.9 \cdot 10^7 \text{ cm / s}$$

$$\rho_{\text{stag}} \approx 1000 \text{ g / cc} \quad \Rightarrow \text{amplification} \sim 4000$$

$$(\rho R)_{\text{stag}} \approx 2 \text{ g / cm}^2 \quad \Rightarrow \text{amplification} \sim 200$$

$$P_{\text{stag}} \approx 500 \text{ Gbar} \quad \Rightarrow \text{amplification} \sim 5000$$

$$E_{\text{Laser}} \sim 1.3 \text{ MJ}$$

Hydrodynamic Scaling Relations from Simulations

<i>Variable</i>	<i>Scaling Relation</i>
Hydrodynamic Efficiency	$\eta \approx \frac{0.051}{I_{15}^{0.25}} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{0.75} \left(\frac{0.35}{\lambda_L (\mu m)} \right)^{0.5}$
Thermonuclear Gain	$G \approx \frac{365}{I_{15}^{0.25}} \left(\frac{3 \times 10^7}{u_{\max} (cm/s)} \right)^{1.25} \left(\frac{\rho R (g/cm^2)}{7 + \rho R} \right) \left(\frac{0.35}{\lambda_L (\mu m)} \right)^{0.5}$
Ignition Energy (MJ)	$E_L^{ign} \approx 0.64 I_{15}^{-0.26} \alpha_{inn}^{1.9} \left(\frac{3 \times 10^7 (cm/s)}{u_{\max}} \right)^{6.6} \left(\frac{\lambda_L (\mu m)}{0.35} \right)$
Shell Areal Density (g/cm ²)	$(\rho R)_{\max} \approx \frac{1.2}{\alpha_{inn}^{0.54}} \left(\frac{E_L (kJ)}{100} \right)^{0.33} \left(\frac{0.35}{\lambda_L (\mu m)} \right)^{0.25} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{0.06}$
Shell Density (g/cm ³)	$\langle \rho \rangle_{\rho R} \approx \frac{425}{\alpha_{inn}^{1.12}} I_{15}^{0.13} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right) \left(\frac{0.35}{\lambda_L (\mu m)} \right)^{0.13}$
Shell IFAR	$IFAR \approx \frac{40 I_{15}^{-0.27}}{\langle \alpha_{IF} \rangle^{0.72}} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{2.12} \left(\frac{\lambda_L (\mu m)}{0.35} \right)^{0.27}$
Hot spot Areal Density (g/cm ²)	$\rho R_h \approx \frac{0.31}{\alpha_{inn}^{0.55}} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{0.62} \left(\frac{E_L (kJ)}{100} \right)^{0.27}$
Hot spot Temperature (keV)	$\langle T_h \rangle \approx \frac{2.96}{\alpha_{inn}^{0.15}} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{1.25} \left(\frac{E_L (kJ)}{100} \right)^{0.07}$
Hot spot Pressure (Gbar)	$\langle P_h \rangle \approx \frac{345}{\alpha_{inn}^{0.90}} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{1.85}$
Stagnation Aspect Ratio	$A_s \approx \frac{1.48}{\alpha_{inn}^{0.19}} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{0.96}$

* Ref. C.D. Zhou and R. Betti PoP paper. With the exception of the gain, all the variables are calculated in the absence of alpha-particle deposition. The gain is calculated by assuming that ignition has taken place.