Российский университет дружбы народов

The Structure of Relativistic Ponderomotive Forces in a Strong Electromagnetic Field

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Contribution

Time-averaged over a field cycle forces acting on charged particles in a spatially inhomogeneous electromagnetic field are usually called Ponderomotive Forces. In the nonrelativistic regime these forces always push the particles away from high-field areas and into the low-field areas. In many cases these forces have a non-relativistic nature. However, in application to electrons, with advent of laser technology, Ponderomotive forces can enter a strongly relativistic domain, where the laser field may readily exceed a relativistic scale $g = eE/mc\omega$. Considering quasi- optical and geometrical optics approximations to handle the Ponderomotive force also relies on adiabatic approximation, whereby one can separate "slow" and "fast" motion to obtain a time-independent "ponderomotive" potential for the "slow" motion proportional to a new relativistic ponderomotive gradient force.

About Ponderomotive Forces

The problem of free (quasi-free) electron (charge) interaction with superstrong electromagnetic fields is gaining great importance due to the progress in laser physics and laser techniques, which made the radiation of intensity available for numerous experiments. In such fields an electron acquires a velocity comparable to the speed of light and, taking relativistic effects into account, becomes fundamentally important. In modern laser physics the concept of ponderomotive forces is used to describe forces averaged over fast variations acting upon a free electron in an inhomogeneous light field. Those forces play a dominant role in laser plasma dynamics (e.g., parametric instabilities, self focusing, beat wave accelerator, fast ignition) In the nonrelativistic approximation, for relatively weak and monochromatic electromagnetic fields of arbitrary space dependence,

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\,\mathbf{E}(\mathbf{r})e^{-i\omega t}, \tag{1}$$

the oscillation center dynamics of a charge motion averaged (ponderomotive drift) is governed by the gradient force [1],

$$\mathbf{F} = -\nabla U(\mathbf{r}) \equiv -\nabla \frac{e^2E^2}{4m\omega^2}, \tag{2}$$

The ponderomotive force is well known to be given by a gradient of the ponderomotive potential where $U(\mathbf{r})$ is the so-called field ponderomotive potential—a scalar function that completely describes to the vector $\mathbf{F}(\mathbf{E},\omega)$ - are the field amplitude and frequency of the inhomogeneous light field under consideration) In the physics of plasma the gradient force (2) is often referred to as the Gaponov–Miller force.

Theoretical Method and Relevant Equations

Equation (2) is a result of a first-order perturbation analysis of the Lorentz force [1]. There were some efforts of relativistic generalization of (2). In [2] the case of charge flying into the laser beam with a large (relativistic) velocity has been investigated. However, the field itself was considered as sufficiently weak. Authors of [3], for calculation of ponderomotive forces, performed the averaging of the relativistic equation of motion on phase $\varphi = \omega t - kr$ (t is the time, r is the charge coordinate, and k is the field wave vector). In [4] the relativistic Lagrangian has been averaged on the same parameter. In many cases it was shown independently by several authors that the ponderomotive force acting on the point charge e is not, in general, a gradient force (that is, it contains a vortex part). Nevertheless, as for weak fields, it still can be described by one scalar function: effective mass m^* of charge in the field. It is found that this force depends on the field polarization, which is important for developing essentially new techniques of charge accelerating or trapping in superstrong fields of combined polarization.

Relativistic Ponderomotive Forces

We calculate the relativistic ponderomotive force (following the Gaponov–Miller approach [1]) by averaging relativistic equation of motion on time in the laboratory frame of reference (L-frame) or on time in the charge oscillation center frame (C-frame), which moves with a velocity of the charge oscillation center relative to L-frame, that is, in the frame where the charge is at rest on average. Such averaging procedure seems to be the most correct one and predicts new features of ponderomotive forces (not revealed by other averaging procedures). The calculations are performed for the case of a sufficiently weak space and time dependent laser envelope without any limits on radiation intensity.

Geometrical optics approximation

Consider radiation with a slightly dependent spatial and temporal envelope, so that the following inequalities are valid $L \gg \lambda;\ T \gg \omega^{-1}$, where λ is the laser wavelength, k is the field wave vector and L, T are the typical space and time scales of the laser envelope. As the charge displacement during the optical cycle (in any field and at any initial charge velocities) is less than λ, these inequalities mean that changes of the field envelope on the interval or while the time are less than changes of field phase. This leads to use $\mu = 1/kL \approx 1/\omega T$ as a parameter of the first order of smallness.

Quasi-optical approximation

In a paraxial beam of radius $r_0 \gg \lambda$, the changes of field polarization are sufficiently small and λ is less than a the beam waist width. This leads to use $\mu = 1/ka$ as a parameter is of the first order of smallness.

References

[1] Gaponov, A.V. and Miller, M.A., 1958, Sov. Phys. JETP, 7, 168.

[2] Bitouk, D.R. and Fedorov, M.V., 1999, Sov. Phys. JETP,89, 640.

[3] Goreslavsky, S.P. and Narozhny, N.B., 1995, J. Nonlinear Opt. Phys. Materials, 4, 799.

[4] Bauer, D., Mulser, P., and Steeb, W.-H., 1995, Phys. Rev. Lett., 75, 4622

With taking into account this regimes, electric and magnetic component of the laser field can be written as

$$\mathbf{E}(\mathbf{r},t) = E_0(\mathbf{r},t)e^{i\theta};\ \ \mathbf{B}(\mathbf{r},t) = B_0(\mathbf{r},t)e^{i\theta}. \tag{3}$$

Where $\mathbf{E}_0(\mathbf{r},t)$, $B_0(\mathbf{r},t)$ describe the plane wave and is a deviation of actual field from the plane wave field in amplitude, phase, and polarization. The phase θ (Eikonal) defines the frequency and wave vector as:

$$\frac{\partial\theta}{\partial t} = -\omega,\ \ \nabla\theta = \mathbf{k} \tag{4}$$

The iteration procedure assumes that all the functions $\mathbf{E}_0, \mathbf{B}_0$ (at condition of slowness of field complex amplitude) are presented in the form of expansions like:

$$\mathbf{E}_0 = \mathbf{E}_{00} + \mu\mathbf{E}_{01} + ...,;\quad \mathbf{B}_0 = \mathbf{B}_{00} + \mu\mathbf{B}_{01} + ... \tag{5}$$

The magnetic and longitudinal components of the field are calculated using consecutive approximation from the Maxwell equations:

$$\mathbf{B}_{00} = \frac{c}{\omega}(\mathbf{k}\times\mathbf{E}_{00})\,;\ \mathbf{B}_{01} - \frac{c}{\omega}(\mathbf{k}\times\mathbf{E}_{01}) = \frac{1}{i\omega}\left(\frac{\partial\mathbf{B}_{00}}{\partial t} + c\nabla\times\mathbf{E}_{00}\right)$$

$$E_{z1} = -\frac{1}{k}\left(\frac{\partial E_{00x}}{\partial x} + \frac{\partial E_{00y}}{\partial y}\right) \tag{6}$$

Let us consider the relativistic Lorentz-force

$$\frac{d\mathbf{p}_\perp}{dt} = e\left(1 - \frac{kv_z}{\omega}\right)\mathbf{E}_0e^{i\theta} - \mu\frac{ekv_z}{i\omega^2}\left(\frac{\partial\mathbf{E}_{00}}{\partial t} + \frac{\omega}{k}\frac{\partial\mathbf{E}_{00}}{\partial z}\right)e^{i\theta} \tag{7}$$

$$\frac{dp_z}{dt} = e\frac{k}{\omega}\mathbf{v}\cdot\mathbf{E}_0e^{i\theta} + \mu e\left(1 - \frac{kv_z}{\omega}\right)E_{1z}e^{i\theta} + \mu\frac{ek}{i\omega^2}\mathbf{v}\cdot\left(\frac{\partial\mathbf{E}_{00}}{\partial t} + \frac{\omega}{k}\frac{\partial\mathbf{E}_{00}}{\partial z}\right)e^{i\theta} \tag{8}$$

in which $\mathbf{p}_\perp$bis the relativistic transversal momentum of the charge and θ is the phase obeying the equation

$$\frac{d\theta}{dt} = -\frac{\omega}{\mu}\left(1 - \frac{kv_z}{\omega}\right) \tag{9}$$

Here v_z is the phase velocity. The parameter μ is introduced to show that phase θ is fast varying.

Strong field with circular polarization

In the case of strong fields with circular polarization, with the help of (9), equation (7) can be transformed to the form in which the electron momentum does not appear explicitly:

$$\pi_x = p_x + \frac{eE}{\omega}\sin\theta,\ \ \pi_y = p_y + \frac{eE}{\omega}\cos\theta \tag{10}$$

Thereby eE/ω is the momentum acquired by the charge in the field. Thus one can obtain the expansion for the Lorentz factor acting on the charge. Let us assume that $\pi_\perp^2 \ll \left(\frac{eE}{\omega}\right)^2$. Here $\pi_\perp^2 = \pi_x^2 + \pi_y^2$.

$$\gamma \approx \gamma_0\left\{1 - \frac{eE}{\omega(mc\gamma_0)^2}(\pi_x\sin\theta + \pi_y\cos\theta)\right\},\ \ \gamma_0 = \left(1 + \frac{1}{(mc)^2}\left\{p_z^2 + \left(\frac{eE}{\omega}\right)^2 + \pi_\perp^2\right\}\right)^{1/2}$$

Under these assumptions we may assume that the momentum can be expanded as $\pi_\perp = \mathbf{P}_\perp + \tilde{\pi}_\perp$, $p_z^2 = P_z^2 + \tilde{p}_z^2$, donde $(\mathbf{P}_\perp, P_z)$, $(\tilde{\pi}_\perp, \tilde{p}_z)$ are slowly and fast varying components. Thus, for slowly varying momentum and velocities we obtain

$$\frac{d\mathbf{P}_\perp}{dt} = -\frac{1}{2\Gamma_0}mc^2\nabla_\perp\left(\frac{eE}{mc\omega}\right)^2;\ \ \frac{dP_z}{dt} = -\frac{k}{2\Gamma_0\omega}mc^2\left(\frac{\partial}{\partial t} + \frac{\omega}{k}\frac{\partial}{\partial z}\right)\left(\frac{eE}{mc\omega}\right)^2 \tag{11}$$

Here Γ_0 is the averaged Lorentz factor. By including the Lorentz factor with respect to the lab time t,

$$\gamma_E = 1 + \left(\frac{eE}{mc\omega}\right)^2 \rightarrow \Gamma_0 = \frac{\gamma_E}{\sqrt{1 - (\vec{\beta}_\perp^2 + \beta_z^2)}} \tag{12}$$

one can obtain the expression for the force acting on the charge. Then averaging this force over fast oscillations. Therefore

$$\frac{d\mathbf{P}_\perp}{dt} = -\sqrt{1 - (\vec{\beta}_\perp^2 + \beta_z^2)}\nabla_\perp m^*c^2,\ \frac{\partial P_z}{dt} = -\frac{k}{\omega}\sqrt{1 - (\vec{\beta}_\perp^2 + \beta_z^2)}\left(\frac{\partial}{\partial t} + \frac{\omega}{k}\frac{\partial}{\partial z}\right)m^*c^2$$

Where $\vec{\beta}_\perp = \frac{\mathbf{P}_L}{mc\Gamma_0}\left(1 - \frac{1}{2}\left(\frac{eE}{mc\omega\Gamma_0}\right)^2\right)$, $\beta_z = \frac{P_z}{mc\Gamma_0}$ and $m^* = m\gamma_E$ is the efective mass of charge in the field. Thus, as for strong fields, ponderomotive forces still are described by one potential function. It is found that this force, in general, depends on the field polarization.

Conclusion

We show a theoretical extension on fully relativistic ponderomotive force acting upon charges in electromagnetic waves. As an example, the general expressions for a ponderomotive force acting upon a relativistic charge in the field with circular polarization are found. These expressions are analyzed for the specific case of a strong laser fields. Under these conditions, the structure of ponderomotive forces is more complex than in the nonrelativistic case and is defined, in general, by all even powers of the electric field amplitude. Nevertheless, for this case, the ponderomotive force still can be described by one scalar function with effective mass m^* depending on the field amplitude E . The "well defined" expression for the averaged Lorentz factor Γ_0 is given.