Collective Focusing of an Intense Ion Beam Propagating Along a Weak Solenoidal Magnetic Field

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Motivation and Outline

- Intense heavy ion beam focusing is of particular importance for High Energy Density Physics and Heavy Ion Fusion Applications.

- Two schemes are considered for intense ion beam focusing.

  - The enhanced radial focusing is provided by the collective dynamics of the plasma electrons.

  - Focusing force is calculated for both approaches.

  - Influence of the collective focusing effects on the ion beam dynamics in a heavy ion driver is discussed.


S. Robertson, PRL 48, 149 (1982).

The enhanced radial focusing is provided by the collective dynamics of the plasma electrons.
Scheme A

Enhanced Self-Focusing of an Ion Beam Propagating Through a Background Plasma Along a Solenoidal Magnetic field

![Diagram showing enhanced self-focusing of an ion beam through a background plasma along a solenoidal magnetic field.]

- Plasma
- Ion beam
- Electron (e−)
- Magnetic field (B0)
- Velocity (V_b)
General Derivation of the Radial Focusing Force

Radial component of the Lorentz force

\[ F_r = Z_b e \left( E_r - \beta_b B_\phi \right) \]

Linear response of the plasma electrons. Steady-state: \( \zeta = z - V_b t \).

Applying the curl operator \((\nabla \times \ldots)\) to Eq. (1)

\[ m_e V_b \frac{\partial V_e}{\partial \zeta} = \frac{e}{c} \left[ V_e \times B_0 \right] + eE \quad (1) \]

Combining the r-component of Eq. (1) with the \( \phi \)-component of Eq. (2)

\[ F_r = Z_b m_e V_b \frac{\partial V_{ez}}{\partial r} \]

Provided the beam current is neutralized, i.e., \( Z_b n_b V_b = n_e V_{ez} \)

\[ F_r = Z_b^2 m_e V_b^2 \frac{1}{n_e} \frac{dn_b}{dr} \]
Enhanced Self-focusing of an Ion Beam

Beam current neutralization condition

\[ r_b \gg r_{ge} \equiv \frac{V_b}{\omega_{ce}} \left( 1 + \omega_{cy}^2 / \omega_{pe}^2 \right)^{1/2} \]

Collective self-focusing

\[ \omega_{ce} \gg \beta_b \omega_{pe} \]

\( \beta_b \sim 0.05, n_p \sim 10^{11} \text{cm}^{-3} \rightarrow B_0 >> 50 \text{ G} \)

Consider the beam radius smaller than the electron skin depth, but greater than the effective electron gyro-radius, \( r_{ge} \).

\[ r_{ge} \ll r_b \ll c/\omega_{pe} \]

The beam current is neutralized.

\[ F_r = Z_b^2 m_e V_b^2 \frac{1}{n_e} \frac{dn_b}{dr} \]

Magnetic self-pinching

\[ B_0 = 0 \]

Consider the beam radius smaller than the electron skin depth

\[ r_b \ll c/\omega_{pe} \]

- The beam current is almost unneutralized.
- The beam charge is well-neutralized.
- Focusing force is produced by the net self-magnetic field.

\[ F_{sp} = Z_b^2 \frac{\omega_{pe}^2}{c^2} m_e V_b^2 \frac{1}{rn_p} \int rn_b \, dr \]

\[ F_{sf} / F_{sp} \sim (c / \omega_{pe} r_b)^2 \gg 1 \]

Self-focusing of an ion beam with \( r_b \ll c/\omega_{pe} \) can be significantly enhanced by application of a weak magnetic field (~100 G).
Numerical PIC Simulations Demonstrate Enhanced Self-Focusing

Gaussian beam: \( r_b = 0.55c/\omega_{pe} \), \( l_b = 3.4r_b \), \( \beta = 0.05 \), \( n_b = 0.14n_p \), \( n_p = 10^{10} \text{ cm}^{-3} \)

The enhanced focusing is provided by a strong radial self-electric field that is generated due to a local polarization of the magnetized plasma background by the moving ion beam.
Local Plasma Response to The Beam is Drastically Different for $\omega_{ce} > 2\beta_b \omega_{pe}$ and $\omega_{ce} < 2\beta_b \omega_{pe}$

Gaussian beam: $r_b = 0.55c/\omega_{pe}$, $l_b = 3.4r_b$, $\beta = 0.05$, $n_b = 0.14n_p$, $n_p = 10^{10}$ cm$^{-3}$

$B_0 = 300$ G ($\omega_{ce}/\beta_b \omega_{pe} = 18.7$)

$B_0 = 25$ G ($\omega_{ce}/\beta_b \omega_{pe} = 1.56$)

Furthermore:

- $\omega_{ce} > 2\beta_b \omega_{pe}$ diamagnetic response ($\delta B_z < 0$)
- $\omega_{ce} < 2\beta_b \omega_{pe}$ paramagnetic response ($\delta B_z > 0$)
- $\omega_{ce} = 2\beta_b \omega_{pe}$ resonant excitation of large-amplitude whistler waves


**Plasma Electron Response in the Presence of a Solenoidal Magnetic Field**

- Due to conservation of the azimuthal canonical momentum, electrons acquire large rotation velocity provided by their radial displacement

\[ mV_{\phi} = e(A_{\phi} + \delta r B_0)/c \]

- External magnetic electron focusing is compensated by the radial electric field

\[ -eB_0 V_{\phi}/c = f_M = f_E = -eE_r \]

\[ \omega_{ce} < 2\beta_b \omega_{pe} \]

Radial electric field is defocusing (for ions)

**Paramagnetic plasma response**

\[ \omega_{ce} > 2\beta_b \omega_{pe} \]

Radial electric field is focusing (for ions)

**Diamagnetic plasma response**
The Enhanced Self-Focusing Can Be Important for Ion Driver Design

The Neutralized Drift Compression Experiment (NDCX)
The heavy ion driver for Warm Dense Matter Experiments

Fringe fields ~100 G penetrate into the long drift section (filled with neutralizing plasma) for distance ~1 m and provide conditions for collective focusing.

Fringe magnetic fields penetrate into background plasma
# Self-Focusing Force Estimates for NDCX-I and NDCX-II

Let's compare the self-focusing and magnetic focusing provided by the final focus solenoid (FFS).

## Self-focusing

\[
F_{self} = m_e V_b^2 \frac{1}{n_p} \frac{dn_b}{dr}
\]

\(\alpha = \omega_{ce}/2\beta\omega_{pe} >> 1\)

### Focusing strength ratio:

\[
\delta = \frac{F_{self}L_d}{F_{sol}L_{FFS}}
\]

- **L\(_d\)** - length of the drift region
- **L\(_{FFS}\)** - solenoid length

<table>
<thead>
<tr>
<th>NDCX-II</th>
<th>NDCX-I</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n_p \sim 10^{11}) cm(^{-3}), (m_i = 7) a.u., (\beta_b = 0.032)</td>
<td>(n_p \sim 10^{11}) cm(^{-3}), (m_i = 39) a.u., (\beta_b = 0.004)</td>
</tr>
<tr>
<td>(\alpha = 1) corresponds to (B = 65) Gs</td>
<td>(\alpha = 1) corresponds to (B = 8) Gs</td>
</tr>
<tr>
<td>(L_{FFS} = 10) cm, (L_d = 200) cm, (r_b = 1) cm</td>
<td>(L_{FFS} = 10) cm, (L_d = 200) cm, (r_b = 1) cm</td>
</tr>
</tbody>
</table>

- **B\(_{FFS}\)** = 8 T
- Assume \(n_b \sim n_p\)
- \(\delta_{NDCX-II} = 0.47\)
- \(\delta_{NDCX-I} = 0.04\)
Enhanced Self-Focusing of an Ion Beam Propagating Through a Background Plasma Along a Solenoidal Magnetic field

Scheme B

diagram:
- Neutralized ion beam (e^-, i^+)
- Magnetic lens
- Background plasma
- Magnetic field (B_0)
Collective Focusing Concept (S. Robertson 1982, R. Kraft 1987)

For the case of a neutralized beam, an ambipolar electrostatic field develops and brings both species to the same focus (experimentally verified).

For a given focal length the magnetic field required for a neutralized beam is smaller by a factor of \((m_e/m_i)^{1/2}\).
The collective focusing concept can be utilized for final ion beam focusing in NDSX.

- No need to fill the final focus solenoid with a neutralizing CAPS plasma.
- Magnetic field of FFS can be decreased from 8 T to ~700 G.
Collective Focusing Lens: Focusing Force

Traversing the region of magnetic fringe fields from $B=0$ to $B=B_0$, electrons and ions acquire angular frequency of

$$\omega_{e,i} = \pm \frac{\Omega_{e,i}}{2} \quad (\Omega_{e,i} = eB_0/m_{e,i}c)$$

Equations of motion

From Robertson (1982)

$$\begin{align*}
\dot{r}_e + \frac{1}{4} r_e \Omega_e^2 + \frac{e}{m_e} E_r &= 0 \\
\dot{r}_i + \frac{1}{4} r_i \Omega_i^2 - \frac{e}{m_i} E_r &= 0
\end{align*}$$

Quasi-neutrality

$$r_e = r_i$$

Focusing Force

$$F_r = eE_r = -\frac{r}{4} m_e \Omega_e^2$$

NDCX-I: $m_i/m_e = 71175$  

8 T FFS can be replaced with 300 G FFS
Conditions for Collective Focusing

- \( \omega_{pe} \geq \Omega_e / \sqrt{2} \) to maintain quasi-neutrality
- \( r_b \leq 2c/\omega_{pe} \) to assure small magnetic field perturbations (due to the beam)

\[
E_r = -\frac{r}{2e} m_e \omega_{pe}^2 \frac{(n_i - n_e)}{n_e} \\
E_r = -\frac{r}{4e} m_e \Omega_e^2
\]

\( \left\{ \begin{array}{l}
\Delta B_z/B_0 = \frac{4\pi e n_e \Omega_e r_b^2}{c 4B_0} = \frac{\omega_{pe}^2 r_b^2}{4c^2} \\
j_\theta = e n_e \Omega_e r / 2
\end{array} \right. 
\]

Neutralizing electrons have to be dragged through the magnetic field fall-off region to acquire necessary rotation (\( \omega_e = \Omega_e / 2 \))

Plasma (or secondary electrons) should not be present inside the FFS, otherwise non-rotating plasma (secondary) electrons will replace rotating electrons (moving with the beam) and enhanced electrostatic focusing will be lost \( \rightarrow \) CAPS must be turned off

Experimentally verified (R. Kraft 1987)
Proof of principle simulation. Idealized case.

**Schematic of the LSP simulation (t=0)**

- Beam density $t=250$ ns
- $n_b = 7\times 10^{12}$ cm$^3$
- Initial beam density of $\sim 10^{10}$ cm$^{-3}$ is required and can be achieved by means of simultaneous drift compression

**Beam injection parameters**

- K$^+$ @ 320 keV
- $n_b = 10^{10}$ cm$^3$
- $r_b = 1$ cm, $T_b = 0.2$ eV
- Beam pulse = 40 ns (5 cm)
- Convergence off; velocity tilt off

**Proof of principle simulation. Idealized case.**

- Initial beam density of $\sim 10^{10}$ cm$^{-3}$ is required and can be achieved by means of simultaneous drift compression
Analytical predictions for the focusing electric field are found to be in agreement with the numerical simulations.
Front-to-End Design of the NDCX Neutralized Beam Simultaneous Compression

Schematic of the simulation

Beam injection parameters:
K\(^{+}\) @ 320 keV, \(r_b=1.6\) cm, \(I_b=27\) mA
\(T_b=0.094\) eV, IBM is on, convergence angle corresponds to 80cm ballistic focal plane.

Plasma parameters (for the simulation II)
\(n_p=10^{11} cm^{-3}\) \(T_e=3\) eV

Current @ 556 cm
(focal plane)

~80X longitudinal compression
**Collective Lens VS Enhanced Self-Focusing**

**Enhanced self-focusing**

\[ F_{\text{self}} = Z_b^2 m_e V_b^2 \frac{1}{n_e} \frac{dn_b}{dr} \]

Condition: \( r_{ge} << r_b << c/\omega_{pe} \)

\[ r_{ge} \equiv \frac{V_b}{\omega_{ce}} \left(1 + \frac{\omega_{ce}^2}{\omega_{pe}^2}\right)^{1/2} \Rightarrow \omega_{ce} >> \beta_b \omega_{pe} \]

**Focusing is robust**

- Persist for arbitrary ratio \( \omega_{ce}/\omega_{pe} \)
- Does not explicitly depend on a local value of the applied magnetic field

\[ F_{\text{col}} = -\frac{r}{4} m_i \Omega_e \Omega_i \]

(\( \Omega_i = Z_b eB_0/m_i c \))

Conditions:

- \( r_b << c/\omega_{pe} \)
- \( \omega_{pe} >> \omega_{ce} \)
- No plasma inside the magnetic lens
- Neutralizing electron should come from the region outside the magnetic field

**In the limit of** \( r_b \sim r_{ge} \) and \( Z_b n_b \sim n_e \) \( \rightarrow F_{\text{self}} \sim F_{\text{col}} \)
Conclusions

Even a weak magnetic field (several hundred gauss) can have a significant influence on neutralized ion beam transport.

Self-focusing force can be significantly enhanced by application of a weak magnetic field (~100 G) for $r_b << c/\omega_{pe}$.

For a given focal length the magnetic field required for a neutralized beam is smaller by a factor of $(m_e/m_i)^{1/2}$.

This effect is important for the design of a heavy-ion driver (e.g. NDCX) and can be utilized for self-pinch ion beam transport applications.

The collective focusing lens can be utilized for an ion beam final focus. The design of the heavy-ion driver (NDCX) utilizing this concept is proposed.