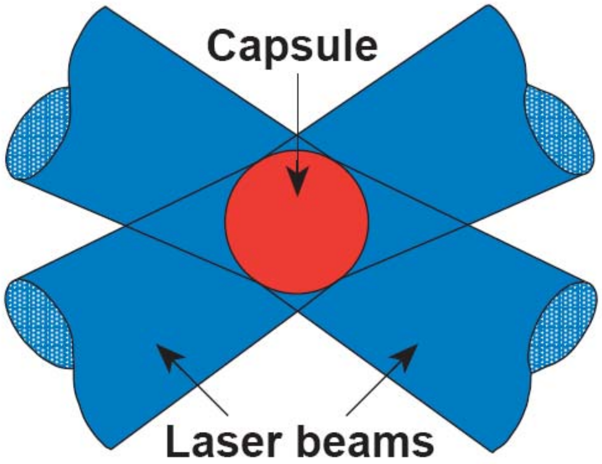
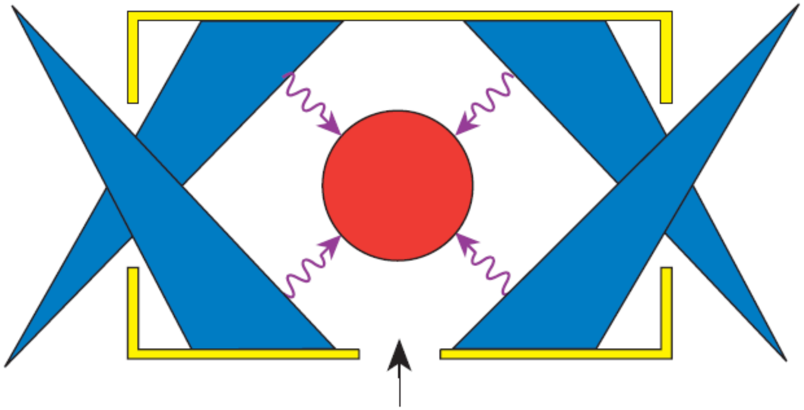
Implosion Hydrodynamics for High Energy-Density Physics and Inertial Confinement Fusion

Direct-drive target



Indirect-drive target



Hohlraum using
a cylindrical high-Z case

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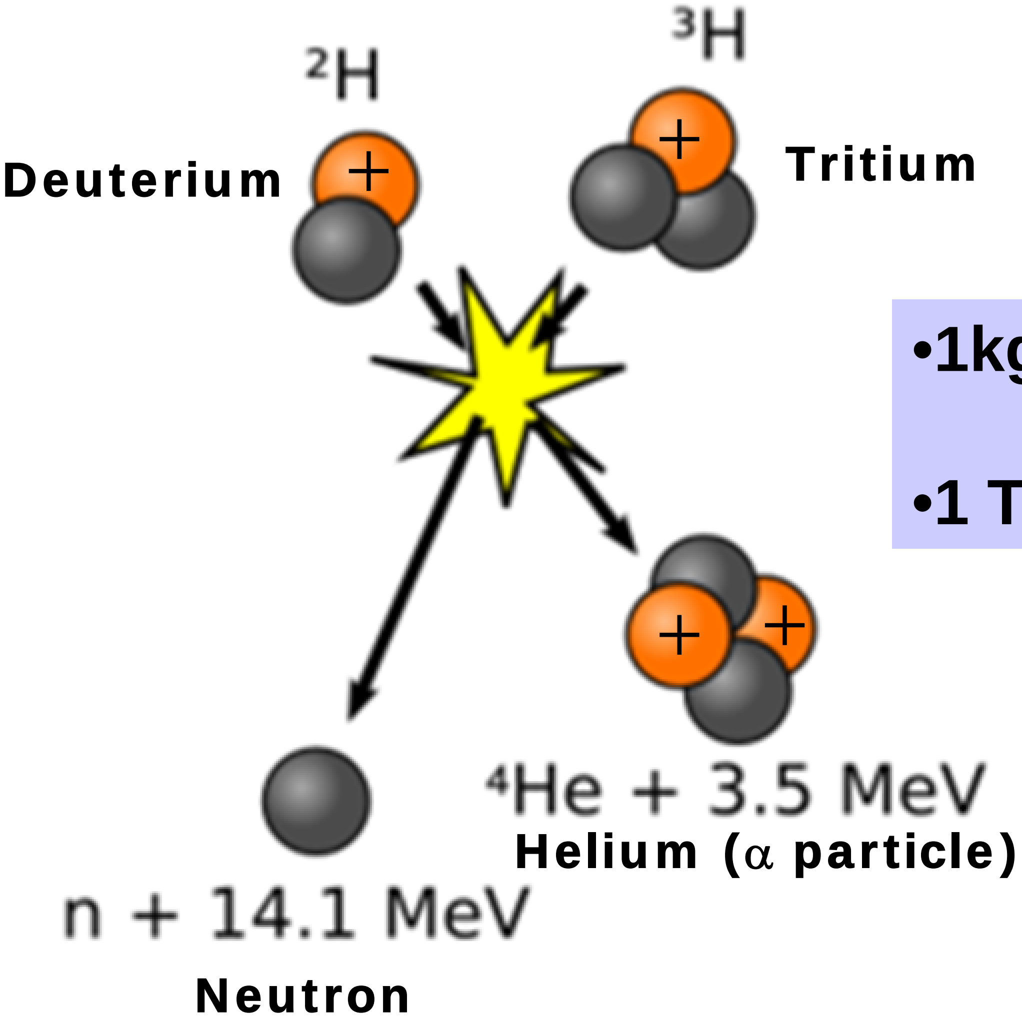
HEDP Summer School, July 27-July 31, 2009, Los Angeles CA

What is nuclear fusion?



- 1kg DT→340 Terajoules
- 1 TJ = 10^{12}J

D + T → α(3.5MeV)+n(14.1MeV)

What could you do with a Terajoule?

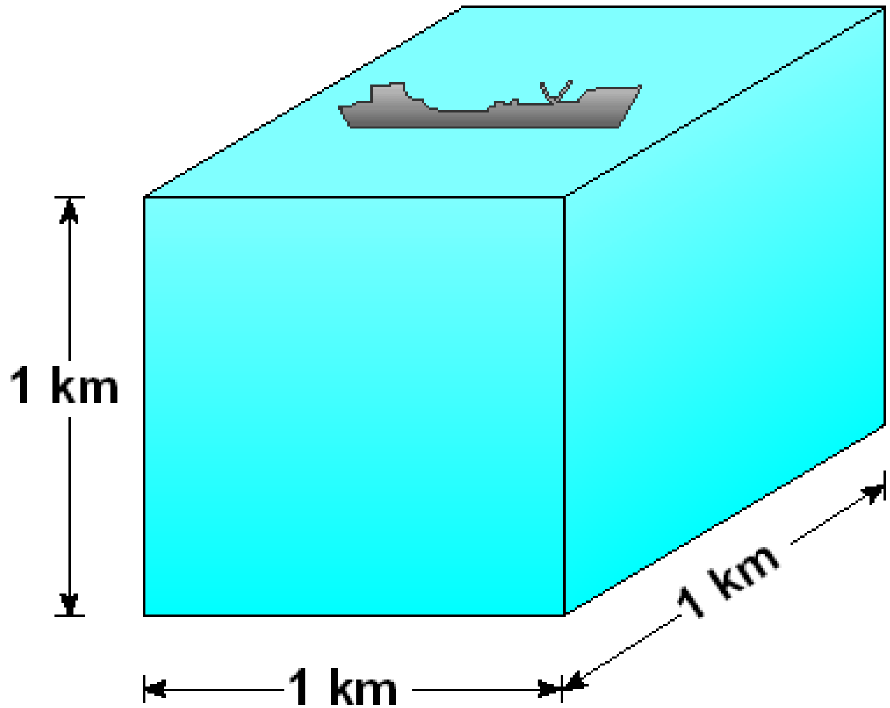
(~3g of DT)

→ **You can drive your car for 625,000 miles**

→ **You can keep your furnace running for 8 years**

→ **You can blow things up! 1TJ = 250 ton of TNT**

Fusion fuel is produced from sea water



= Total energy of world oil reserve

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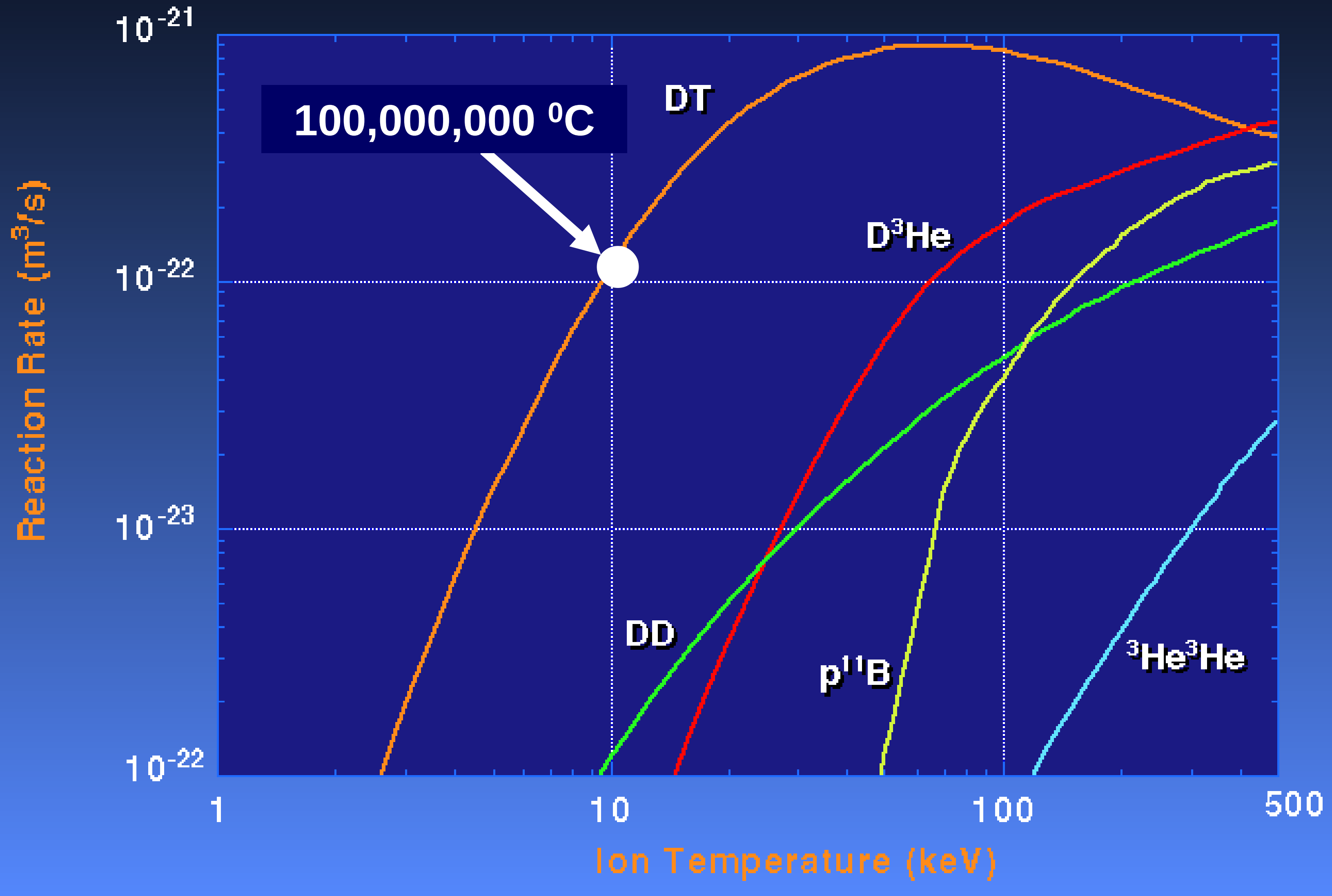
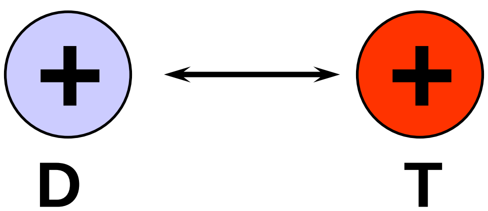
Fusion doesn't come easy

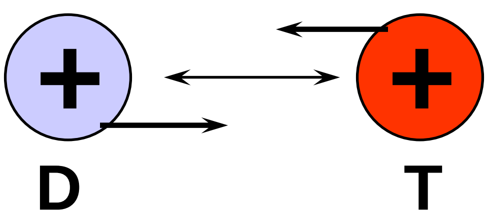


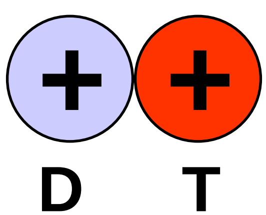
A "hot plasma" at 100M ^{0}C is needed

- Probability for fusion reactions to occur is low at low temperatures because of Coulomb repulsion force.



- If the ions are sufficiently hot (i.e. large random velocity) then they can collide by overcoming Coulomb repulsion

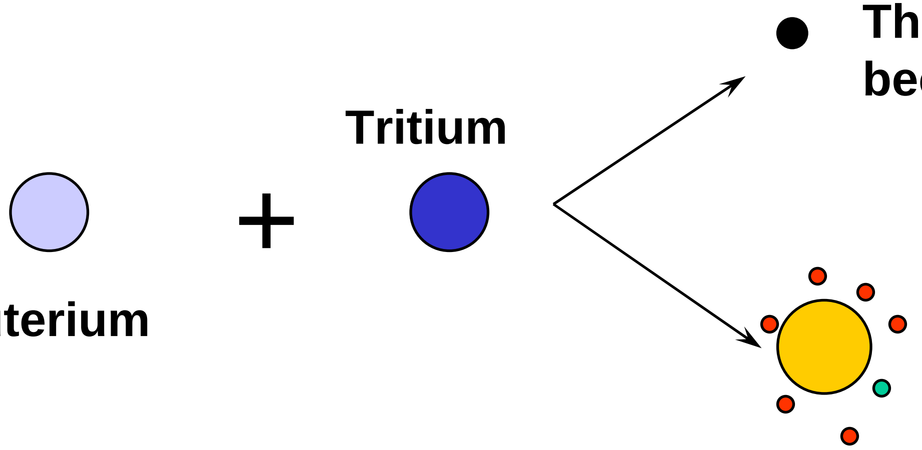




Doesn't it take a lot of energy to keep the plasma at 100M ^{0}C? YES!

SOLUTION:

→Let the plasma do it itself!



THE α-PARTICLES HEAT THE PLASMA!

Under what conditions the plasma keeps itself hot?

Ignition condition

$P\,\tau > 10$ atm-sec

P= pressure in atmospheres

τ= confinement time in seconds

The plasma is too hot to be kept inside a solid container

SOLUTIONS:

 # 1→Use a magnetic field to contain it.
Since the plasma is made of charged particles (ions + electrons), a magnetic field should confine it.

Magnetic Confinement Fusion
P~atm, τ~sec, T~10^8 ^oC

#2→Don't confine it!
Or you can say that the plasma is confined by its own inertia

Inertial Confinement Fusion
P~Gigabar, τ~nanosecond, T~10^8 ^oC

The high pressures and temperatures required for inertial fusion can be achieved through **laser-driven spherical implosions** of a thin shell

Laser-driven spherical implosions can be used to:

→ **Achieve extreme states of matter of interest for inertial confinement fusion and/or general HEDP**

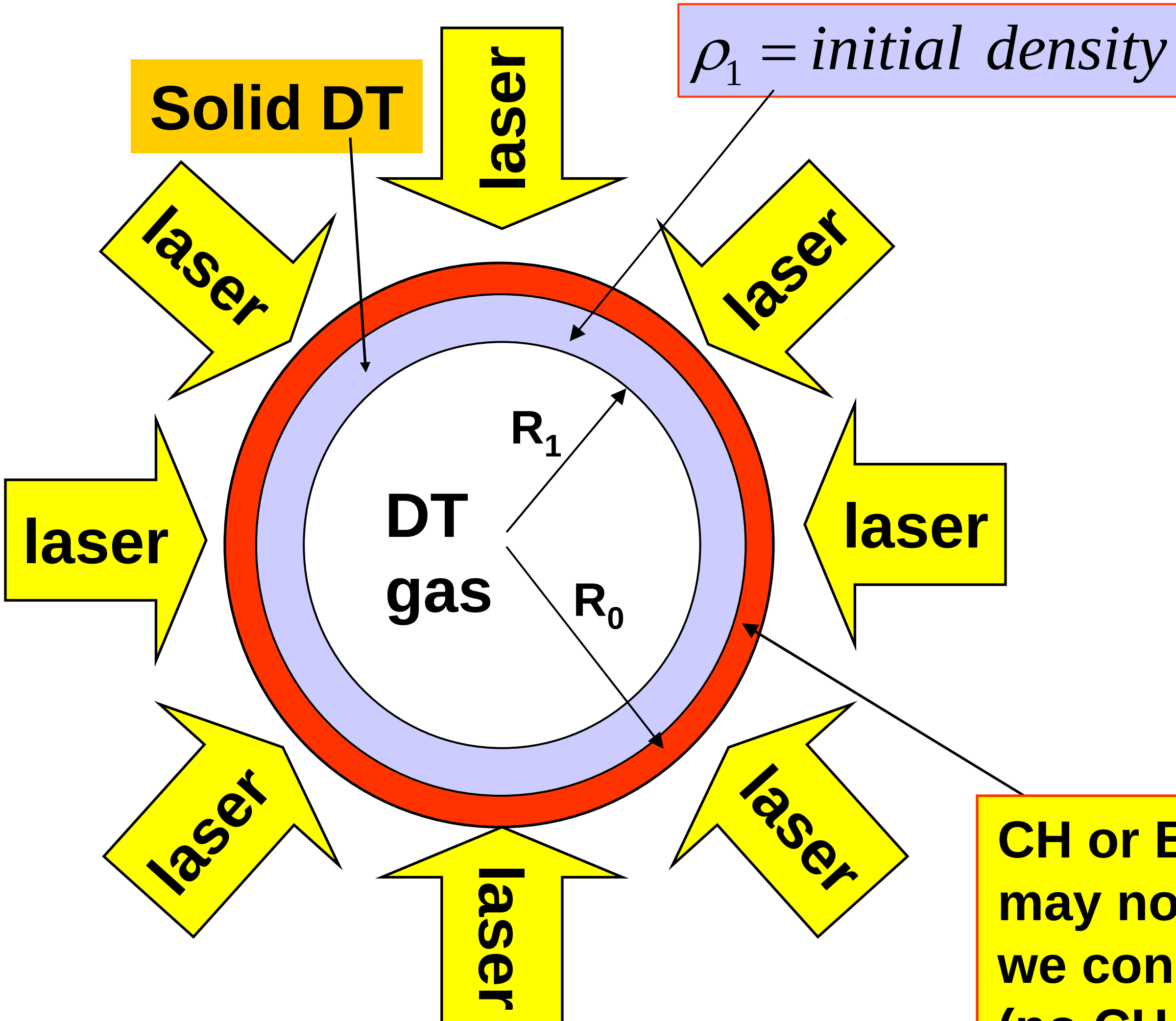
→ **Achieve high temperatures (~ 10^{8-9} ^{o}C→10-100keV)**

→ **Achieve high densities (~300-1000g/cc)**

→ **Achieve high pressures (~10^{9-12} atm→Gbar-Tbar)**

→ **Achieve high areal densities =ρR (~1-3g/cm^2)**

Laser-driven imploding capsules are mm-size shells with hundreds of μm thick layers of cryogenic solid DT



Cryogenic solid DT ice at 18K

$$\rho_1 = 0.25 g / cc$$

$$\rho_{gas} = 5.6 \bullet 10^{-4} g / cc$$

$$p_{gas} \approx 1 atm$$

$$R_1 \approx 1.7 mm$$

$$R_0 \approx 1.3 mm$$

CH or Be ablator may or may not be present. Here, we consider DT targets only (no CH or Be)

LECTURE # 1

Fundamentals of implosion hydrodynamics

We will use the conservation equations of gas-dynamics + ideal gas EOS to treat the DT plasma

$$\partial_t \rho + \partial_x (\rho u) = 0$$

←**mass conservation**

$$\partial_t (\rho u) + \partial_x \left(p + \rho u^2\right) = \text{body forces}\,(\text{ex}: \rho\vec{g})$$

←**momentum conservation**

$$\partial_t \varepsilon + \partial_x \left[v(\varepsilon + p) - \kappa \partial_x T\right] = \text{sources} + \text{sinks}$$

←**energy conservation**

$$p = pressure = (n_e T_e + n_i T_i) = 2nT = (2/m_i)\rho_i T$$

←**ideal gas equation of state**

$T_e = T_i = T = i/e$ temperature

$n_e = n_i = n = i/e$ particle density

(for DT plasmas n_e=n_i)

$$\varepsilon = \frac{3}{2} p + \rho \frac{u^2}{2}$$

←**Total energy per unit volume**

$\rho = \text{mass density} = \mathrm{n_i m_i}$

κ = plasma thermal conductivity

$u = \text{velocity}$

The plasma thermal conductivity goes like a power law of T

$$n\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\kappa\frac{\partial T}{\partial x}$$

Dimensional analysis →

$$n\frac{T}{t} \sim \frac{\kappa T}{x^2} \Rightarrow \kappa \sim n\frac{x^2}{t}$$

$$x \Rightarrow \lambda_{mfp} \sim \upsilon_{th}\tau_{coll} \sim \frac{\upsilon_{th}}{\nu_{coll}}$$

$$t \Rightarrow \tau_{coll} = \frac{1}{\nu_{coll}}$$

$$\kappa \sim n\frac{\upsilon_{th}^2}{\nu_{coll}} \qquad \upsilon_{th}^2 \sim \frac{T}{m_e} \qquad \nu_{coll} \sim \frac{n}{T^{3/2}} \Rightarrow \kappa \sim T^{5/2}$$

υ_{th} = thermal velocity

ν_{coll} = collision frequency

τ_{coll} = collision time

Plasma thermal conductivity

$$\kappa \approx \kappa_0 T^{5/2}$$

Sound speed in an ideal DT gas/plasma

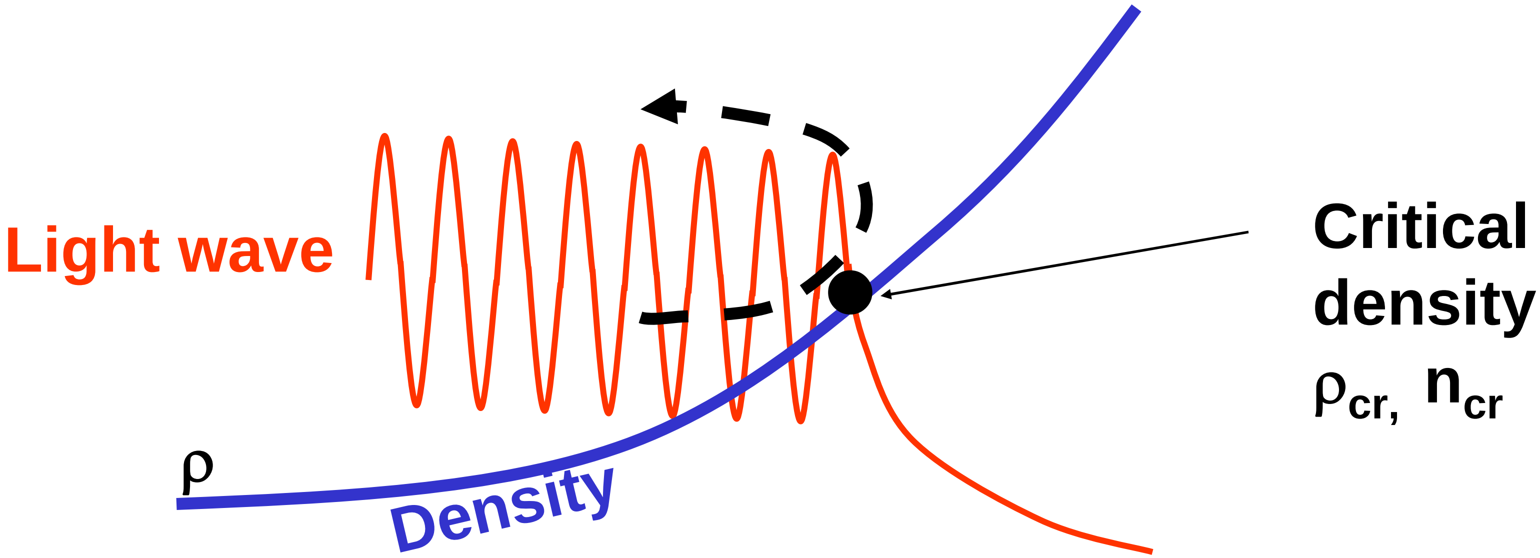
Adiabatic sound speed when the entropy is conserved along the fluid motion

$$C_s^{adiabatic} = C_s(\text{constant entropy}) = \sqrt{\frac{5}{3}\frac{p}{\rho}} = \sqrt{\frac{10}{3}\frac{T}{m_i}}$$

Isothermal sound speed when the temperature is constant along the fluid motion

$$C_s^{isothermal} = C_s(\text{constant temperature}) = \sqrt{\frac{p}{\rho}} = \sqrt{\frac{2T}{m_i}}$$

The laser light cannot propagate past a critical density (see LPI lectures)



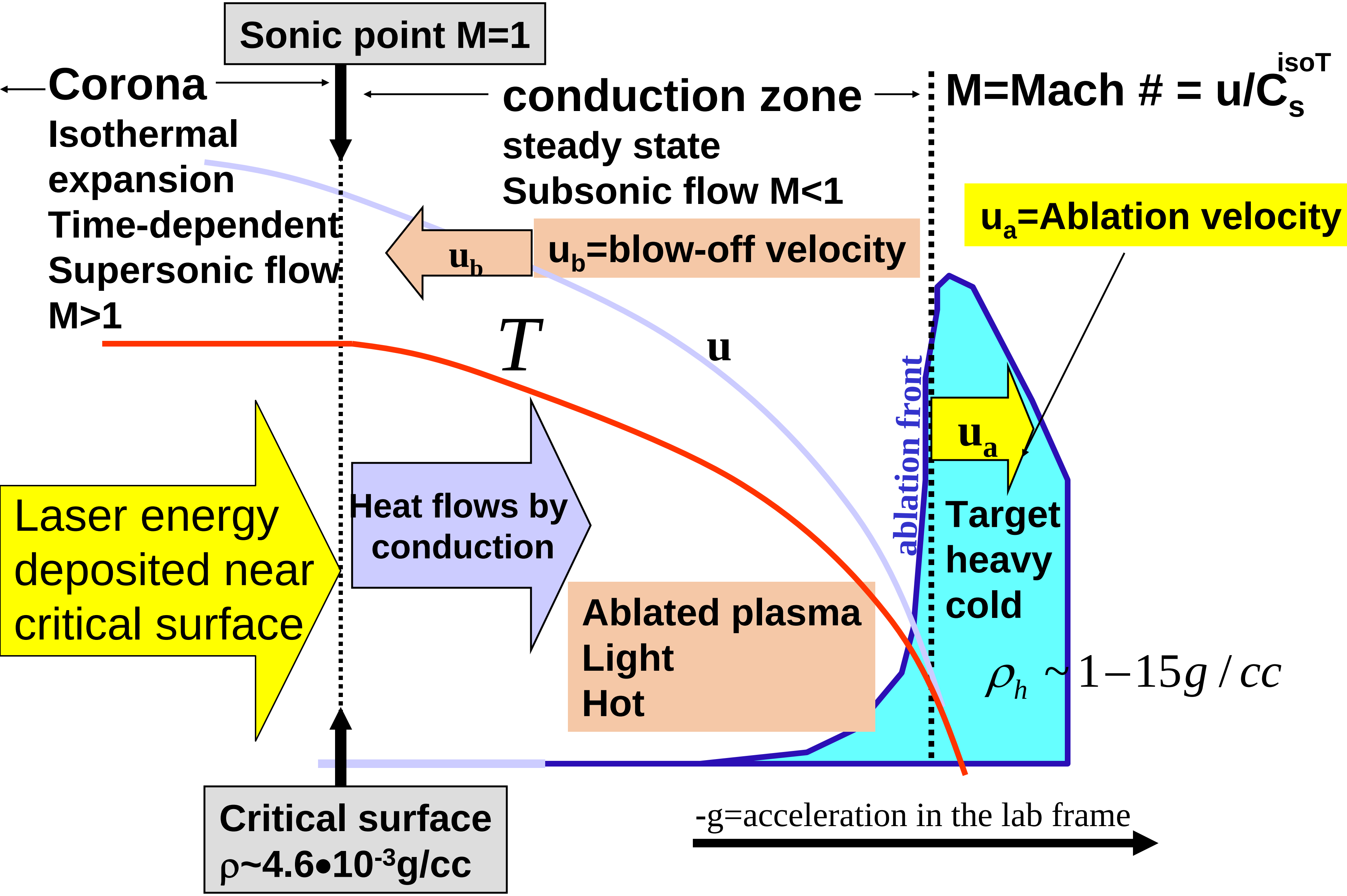
- **Critical density given by: plasma frequency= laser frequency**

$$\omega_L^2 = \omega_{pe}^2 \qquad \omega_L = \frac{2\pi c}{\lambda_L} \qquad \omega_{pe} = \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

$$n_e^{cr} = \frac{1.1 \bullet 10^{21}}{\lambda_L(\mu m)^2} cm^{-3}$$

$$\rho_{DT}^{cr} = \frac{4.6 \bullet 10^{-3}}{\lambda_L(\mu m)^2} g/cc$$

The laser generates a pressure by depositing energy at the critical surface



What is the pressure generated by the laser?

• **Use the energy conservation equation**

$$\partial_t \varepsilon + \partial_x \left[u(\varepsilon + p) - \kappa \partial_x T \right] = \underbrace{I\delta(x - x_c)}$$

Energy deposited by the laser near critical W/cm^{3}s

• **Since the temperature gradients are small in the corona, the heat flux is small**

$$\kappa \partial_x T(x \geq x_c) << \kappa \partial_x T(x \leq x_c)$$

More accurately

$$\kappa \partial_x T(x \geq x_c) \approx \frac{1}{3} \kappa \partial_x T(x \leq x_c)$$

• **Integrate around critical surface x_c**

$$-\left[\kappa \partial_x T \right]_{x_c^-}^{x_c^+} = I \quad \Longrightarrow \quad \kappa \partial_x T(x_c^-) = I$$

• **Solving at steady state in the conduction zone ($x<x_c$) leads to**

$$v(\varepsilon + p) \sim \kappa \partial_x T \quad \text{for } \; x \leq x_c^-$$

- **At the sonic point (i.e. critical surface)**

$$I = \left[u(\varepsilon + p)\right]_{x_c^-} = c_s\left(\frac{5}{2}p_c + \rho_c \frac{C_{s(isoT)}^2}{2}\right) \sim \frac{p_c^{3/2}}{\rho_c^{1/2}}$$

- **The total pressure (static + dynamic) is the ablation pressure**

$$P_A = [p + \rho u^2]_{x=x_c} = 2p_c \sim \left(I\rho_c^{1/2}\right)^{2/3} \sim \left(\frac{I}{\lambda_L}\right)^{2/3}$$

- **The laser-produced total (ablation) pressure on target is**

$$P_A \approx 83Mbar\left(\frac{I_{15}}{\lambda_{L(\mu m)}/0.35}\right)^{2/3}$$

I_{15} laser intensity in 10^{15} W/cm^2

$\lambda_{L(\mu m)}$ laser wavelength in microns

What is the mass ablation rated induced by the laser?

- **At steady state, the mass flow across the critical surface must equal the mass flow off the shell (i.e. the mass ablation rate $\dot{m}_a$)**

$$\dot{m}_a = \rho u = \rho_c u_c = \rho_c C_{s(isoT)}^{crit} = \rho_c \sqrt{\frac{p_c}{\rho_c}} = \sqrt{\rho_c p_c}$$

$$\rho_c \sim \frac{1}{\lambda_L^2} \qquad p_c \sim \left(\frac{I}{\lambda}\right)^{2/3} \quad \Rightarrow \quad \dot{m}_a = \frac{I^{1/3}}{\lambda_L^{4/3}}$$

$$\dot{m}_a = 3.26 \bullet 10^5 \frac{I_{15}^{1/3}}{\lambda_{L(\mu m)}^{4/3}} g/cm^2 s$$

What is the entropy of an ideal gas/plasma?

- **The entropy S is a property of a gas just like P, T and ρ**

$$S = c_v \ln\left[\frac{p}{\rho^{5/3}} const\right] = c_v \ln[\alpha]$$

$$\alpha = const \frac{p}{\rho^{5/3}}$$

- **We call α the "adiabat"**

- **The entropy/adiabat S/α changes through dissipation or heat sources or sinks**

$$\rho\left(\frac{\partial S}{\partial t} + \vec{u} \bullet \nabla S\right) = \frac{DS}{Dt} = \mu \frac{\left|\nabla \vec{u}\right|^2}{T} + \frac{\nabla \bullet \kappa \nabla T}{T} + \text{sources/sinks}$$

- **In an ideal gas (no dissipation) and without sources and sinks, the entropy/adiabat is a constant of motion of each fluid element**

$$\frac{DS}{Dt} = 0 \Rightarrow S, \alpha = const \Rightarrow p \sim \alpha \rho^{5/3}$$

A low adiabat (entropy) gas is easy to compress

- **smaller α → less work to compress from low to high density**

$$W_{1\to 2} = -\int_{\rho_1}^{\rho_2} p dV \sim -\int_{\rho_1}^{\rho_2} \alpha\rho^{5/3} d\frac{M}{\rho} \sim \alpha M\left(\rho_2^{2/3} - \rho_1^{2/3}\right)$$

- **smaller α → higher density for the same pressure**

$$\alpha \sim \frac{P}{\rho^{5/3}} \qquad \Rightarrow \qquad \rho \sim \left(\frac{\mathrm{P}}{\alpha}\right)^{3/5}$$

- **in HEDP, the constant in the definition of the adiabat comes from the normalization of the pressure with the Fermi pressure**

$$\alpha \equiv \frac{p}{p_F} \qquad \Rightarrow \text{(for DT plasma)} \qquad \alpha_{DT} \equiv \frac{p(Mb)}{2.2\rho(g/cc)^{5/3}}$$

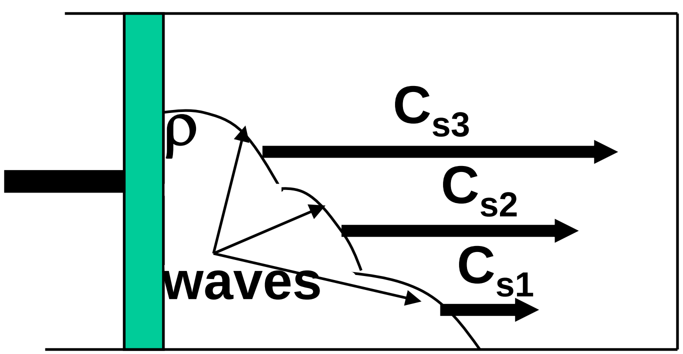
What is a shock?

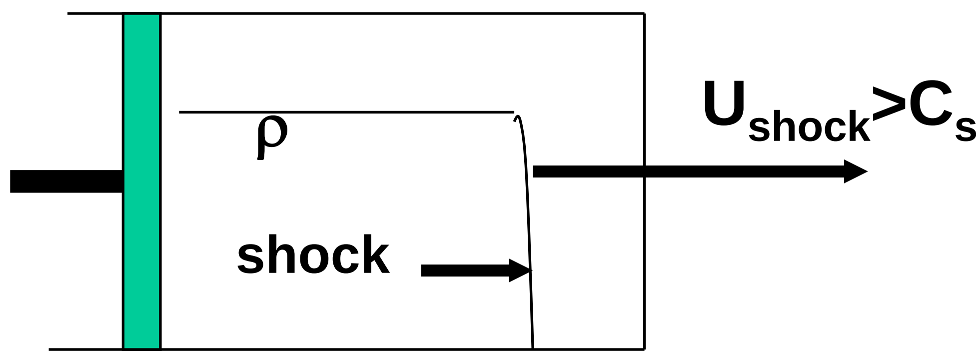
- If a gas/plasma is rapidly compressed by a piston, the acoustic/compression waves launched by the piston overlap due to the always increasing sound speed of a compressed gas/plasma. This overlap causes a steepening of the hydro properties→SHOCK

$$C_s \sim \sqrt{\frac{p}{\rho}} \sim \sqrt{\frac{\alpha\rho^{5/3}}{\rho}} \sim \sqrt{\alpha}\,\rho^{1/3}$$

Sound speed increases with density

$\tilde{\rho}_1$



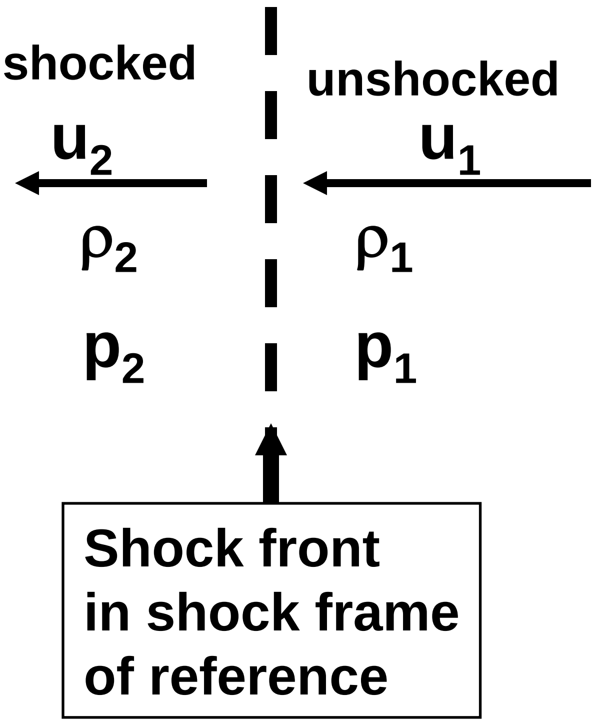


The flow of mass, momentum and energy is conserved across the shock front →Rankine-Hugoniot conditions

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$u_1(\varepsilon_1 + p_1) = u_2(\varepsilon_2 + p_2)$$



- **For an ideal gas/plasma**

$$\varepsilon = \frac{3}{2}p + \rho\frac{u^2}{2}$$

- **For example: assign, ρ_1, p_1 and p_2 to find ρ_2, u_2, $u_1 = -U_{shock}$ using the three R-H conditions**

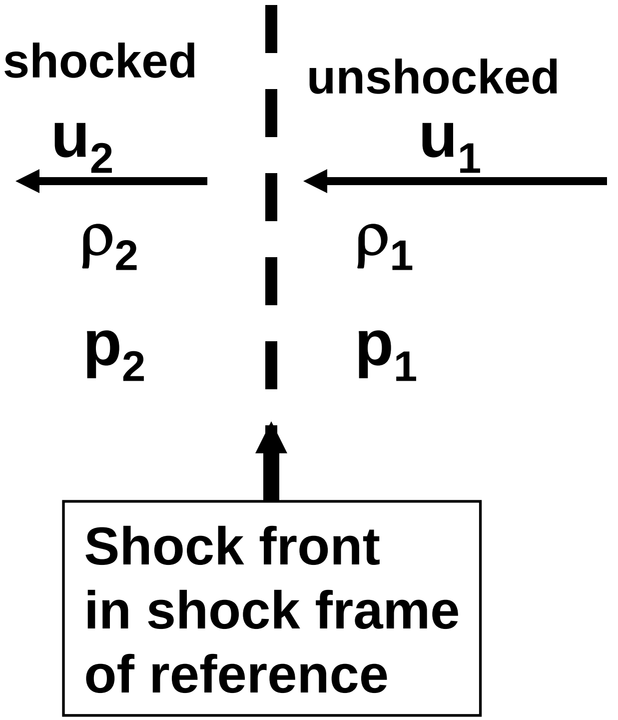
For a strong shock ($p_2 >> p_1$) the R-H are simplified

$$\frac{\rho_2}{\rho_1} \approx 4$$

$$U_{shock} = -u_1 \approx \sqrt{\frac{4p_2}{3\rho_1}}$$

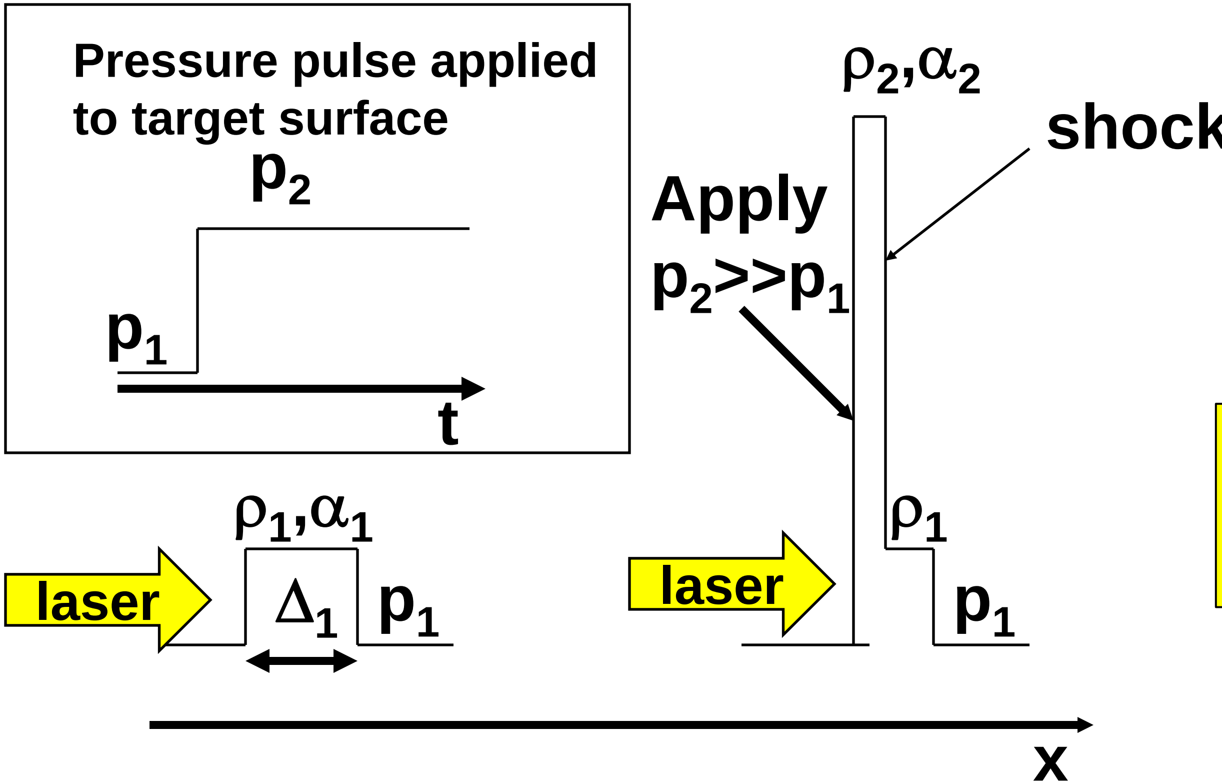
$$u_2 \approx \sqrt{\frac{p_2}{12\rho_1}}$$

$$\frac{\alpha_2}{\alpha_1} = \frac{p_2 / \rho_2^{5/3}}{p_1 / \rho_1^{5/3}} \approx \frac{1}{4^{5/3}} \frac{p_2}{p_1} >> 1$$



The entropy/adiabat increases through the shock

In an ideal gas/plasma, the adiabat α is constant unless a shock is present that raises the adiabat



Post-shock density

$$\rho_2 \approx 4\rho_1$$

Adiabat set by the shock for DT

$$\alpha_2 \approx \frac{p_2(Mb)}{2.2\left(4\rho_1(g/cc)\right)^{5/3}}$$

- **Time required for the shock to reach the rear target surface (shock break-out time = t_{sb})**

$$t_{sb} = \frac{\Delta_1}{u_{shock}} = \Delta_1\sqrt{\frac{3\rho_1}{4p_2}} \propto \sqrt{\frac{1}{\alpha_2 \rho_1^{5/3}}}$$

• **If the target is initially cryogenic solid DT at 18K, then**

$$\rho_1 = 0.25 g / cc \qquad \alpha = \frac{p_{foot}(Mbar)}{2.2} \qquad P_{foot} \approx 83 Mbar \left(\frac{I_{15}^{foot}}{\lambda_{\mu m} / 0.35} \right)^{2/3}$$

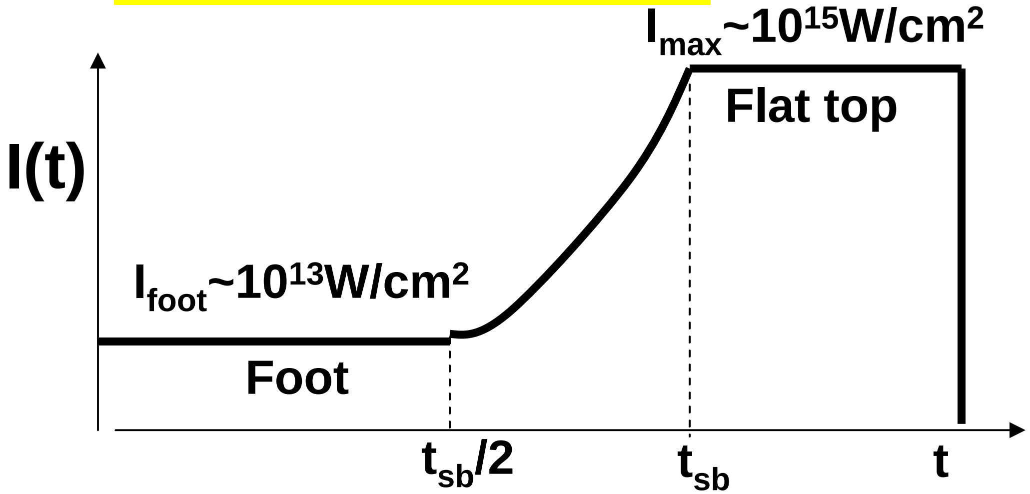
$$I \approx 4.3 \bullet 10^{12} \frac{W}{cm^2} \Rightarrow p_{foot} = 2.2 Mbar \Rightarrow \alpha = 1$$

$$I \approx 1.2 \bullet 10^{13} \frac{W}{cm^2} \Rightarrow p_{foot} = 4.4 Mbar \Rightarrow \alpha = 2$$

$$I \approx 2.2 \bullet 10^{13} \frac{W}{cm^2} \Rightarrow p_{foot} = 6.6 Mbar \Rightarrow \alpha = 3$$

In order to accelerate a shell to high velocity without raising the adiabat, the pressure must be "slowly" increased after the first shock.

Laser pulse shape



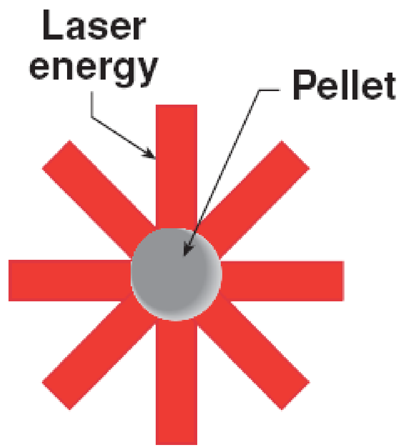
t_{sb} is the shock break-out time

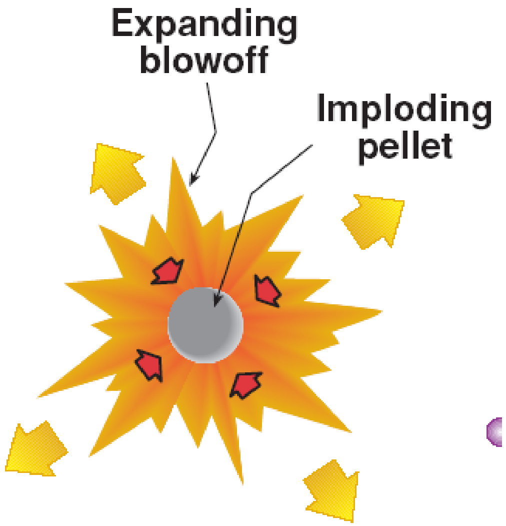
I(t) is the time dependent laser intensity

- After the foot of the laser pulse, the laser intensity must be raised starting at about $0.5t_{sb}$ and reach its peak at about t_{sb}

- Reaching I_{max} at t_{sb} prevents a rarefaction/decompression wave to propagate back from the rear target surface and decompress the target.

Most of the laser energy absorbed by the plasma goes into the kinetic and thermal energy of the expanding blow-off plasma rather than into kinetic energy of the imploding shell





$$M\frac{du}{dt} = -4\pi R^2 P_a$$

Shell Newton's law

$$\frac{dM}{dt} = -4\pi R^2 \dot{m}_a$$

Shell mass decreases due to ablation

$$P_a = \dot{m}_a u_{exhaust}$$

Ablation pressure = Abl. Rate X exhaust vel.

The rocket model

Integrating the rocket equations yields the shell velocity, the shell mass and the hydro efficiency that depends on the ablated mass.

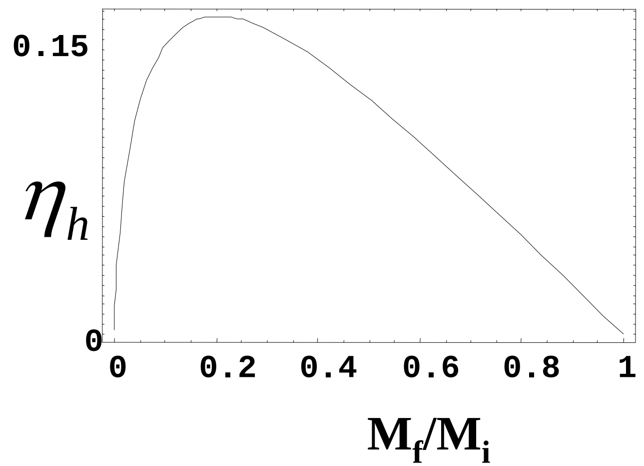
- **Assume that driver (i.e. P_a) is on till the shell is about ½ initial radius**

$$u_{shell} = u_{ex} \ln\left(M_{initial} / M_{final}\right)$$

$$E_{kinetic}^{shell} = M_{final}\left[u_{ex} \ln\left(M_{initial} / M_{final}\right)\right]^2 / 2$$

$$E_{exhaust} = (M_{initial} - M_{final})\left(u_{ex}^2 / 2 + 3P_{ex} / 2\rho_{ex}\right)$$

$$\text{Take } u_{ex}^2 \approx u_{cr}^2 \approx C_{s(isoT)}^2 \approx P_{ex} / \rho_{ex} \approx P_{cr} / \rho_{cr}$$



Hydro efficiency

$$\eta_h = \frac{E_{kinetic}^{shell}}{E_{exhaust}} = \frac{M_f / M_i\left[\ln(M_f / M_i)\right]^2}{4\left(1 - M_f / M_i\right)}$$

$$M_{initial} - M_{final} = M_{ablated}$$

Efficiency is maximum for M_{final}~$0.2M_{initial}$. This is not a good operating point for a DT ablator (Direct Drive) but works for a non-DT ablator (Indirect Drive) For Direct Drive η_h~8-10%.

Homework problem

- Design a direct-drive laser pulse for a cryogenic DT shell with inner radius R_1=1.35mm and outer radius R_0=1.7mm. Plot the laser power in Terawatts (10^{12}W/cm^2) versus time in nanoseconds (10^{-9} s).
- The laser always shines on the outer surface at $r=R_0$
- The total pulse energy is 1.5MJ
- The initial DT density is 0.25g/cc.
- The maximum laser intensity is 10^{15} W/cm^2

Homework problem (continue)

- The foot of the laser pulse must set the shell on an adiabat α=3
- Calculate the shock break-out time t_{sb}
- Use a cubic power law in time to raise the intensity from $0.5t_{sb}$ to t_{sb}
- For a UV laser with λ_L=0.35μm, estimate the ablation pressure at I_{max} , the fraction of ablated mass and the hydro-efficiency
- Assuming 60% of laser energy absorption, estimate the final shell implosion velocity