Hot spot dynamics in 1D spherical implosions

- Converging shock
- Imploding shell
- Low density plasma
- Shock-heated low-density plasma
- Diverging return shock
- Adiabatic compression-heated “hot spot”
The decelerating slab problem provides the basic understanding of the deceleration phase and hot spot formation.

- The shock reflected from the wall slows down the foil, which in turn compresses the gas and decelerates.
- The 1-D problem can be solved analytically leading to a clear understanding of the relevant physics issues.
The low density gas is heated to form a hot spot.
The heat flux leaving the hot spot is deposited onto the shell surface causing mass ablation from the shell into the hot spot. The hot spot mass increases in time.

\[ q_{heat} = -\kappa(T)\nabla T \]

\[ \kappa(T) \approx \kappa_0 T^{5/2} \]

The heat leaving the hot spot cannot penetrate the shell because the shell is cold and its thermal conductivity is low,

\[ \kappa_{shell} \ll \kappa_{hot \text{ spot}} \]

The heat is deposited on the shell inner surface causing mass ablation off the shell.
\[ u_b = \text{blow-off velocity into hot spot} \]

\[ \frac{5}{2} p u_b = -\kappa_0 T_0^{5/2} \frac{dT}{dR_{hs}} \]

- Hot spot temperature profile: \[ T_{hs} = T_0 \left(1 - \frac{r^2}{R_{hs}^2}\right)^{2/5} \]

- Use ideal gas EOS: \[ p u_b = 2 \rho_{R_{hs}} T_{R_{hs}} u_b / m_i = 2 m T_{R_{hs}} / m_i \]

- Ablation rate into hot spot: \[ m = \rho_{R_{hs}} u_b \]

\[ m = 0.2 \frac{m_i \kappa_0 T_0^{5/2}}{R_{hs}} \]
Hot spot density

$M_{hs} = \rho_{hs} V_{hs} = \frac{m_i}{2} \frac{pV_{hs}}{T_{hs}}$

Mass

$\frac{dM_{hs}}{dt} = 4\pi R_{hs}^2 \dot{m}$

Mass conservation

$t \sim \frac{R_{hs}}{u_{max}}$

Hot spot compression time

$pV_{hs} \sim M_{shell} u_{max}^2$

Energy Conservation

(.hot spot internal energy comes from shell kinetic energy)

Hot spot volume $\sim R_{hs}^3$

Use EOS $\rho = m_i p / 2T$

Hot spot mass

$\rho_{hs} = \rho_0 u_{max}$

Hot spot temperature

$T \sim \left( \frac{M_{shell} u_{max}^3}{\kappa_0 R_{hs}^2} \right)^{2/7} \sim \frac{1}{\kappa_0} \left[ \left( \rho R \right)_{stag} \right]^{2/7} u_{max}^{6/7}$

Shell areal density

$\rho_{shell} \sim \rho R^3$
- Use the areal density scaling found in previous lecture

\[
(\rho R)_{\text{shell}}^{\text{stag}} \approx \frac{E_L^{1/3} u_{\text{max}}^{2/3} I^{4/45}}{\alpha^{4/5}}
\]

- Find the temperature scaling. The hot spot temperature mainly depends on the implosion velocity.

\[
T_{\text{hot-spot}} \approx \frac{E_L^{0.1} I^{0.03}}{\alpha^{0.2}} u_{\text{max}}^{1.1}
\]

- Simulation results (without \(\alpha\)-heating) confirm the theory

\[
\langle T_{\text{hot-spot}} \rangle (\text{keV}) \approx \frac{3}{\alpha^{0.15}} \left( \frac{u_{\text{max}} (\text{cm/s})}{3 \times 10^7} \right)^{1.25} \left( \frac{E_L (\text{kJ})}{100} \right)^{0.07}
\]
HYDRODYNAMIC INSTABILITIES

a = acceleration in the lab frame

g = -a = acceleration in the shell frame

Imploding dense shell

Outer shell surface

Inner shell surface

Dense → heavy

Low density → light

In the shell frame

Low density ablated expanding plasma

laser
THE CLASSICAL RAYLEIGH-TAYLOR INSTABILITY
of
A HEAVY FLUID SUPPORTED BY A LIGHTER FLUID

EQUILIBRIUM CONDITIONS

Pressure gradient is opposite to density gradient

\[ \frac{dP}{dx} = \rho_h g \]

\[ \frac{dP}{dx} = \rho_\ell g \]

\( \rho_h \)

\( \rho_\ell \)

heavy

light

\( g \)

\( x \)
The classical R-T is just Newton’s law at work: $F = ma$

$$F = S(P_h - P_\ell) = ma = \rho_h \lambda S \ddot{\eta}$$

$$\frac{dP_0}{dx} = \rho_0 g = \begin{cases} \rho_h g & \text{heavy} \\ \rho_\ell g & \text{light} \end{cases}$$

$$P_\ell = P_0 + \left[ \frac{dP_0}{dx} \right]_\ell \ddot{\eta} = P_0 + \rho_\ell g \ddot{\eta}$$

$$P_h = P_0 + \left[ \frac{dP_0}{dx} \right]_h \ddot{\eta} = P_0 + \rho_h g \ddot{\eta}$$

$$k \sim 1/\lambda$$

$$F = ma \rightarrow S \rho_h g \ddot{\eta} = \rho_h \lambda S \dddot{\eta} \rightarrow \dddot{\eta} = kg \ddot{\eta} \rightarrow \ddot{\eta} \sim e^n \rightarrow \gamma = \sqrt{kg}$$
In the laser irradiated targets the heat/ablation front penetrates at the ablation velocity $u_a$. The heat/ablation front penetrates at the ablation velocity $u_a$. The ablation velocity $u_a$ is given by $u_a = \frac{\dot{m}_a}{\rho_h}$, where $\dot{m}_a = \rho_h u_a = \rho_\ell u_b$.

The blow-off velocity $u_b$ is equal to the acceleration in the target frame $g = -a$. The critical surface is defined by $\rho = \rho_\ell$ and the heat flows by conduction. The M<1 subsonic flow is shown. The M>1 supersonic flow is not shown. The Laser Energy deposited is shown on the left side of the diagram. The Corona is shown. The ablation front is shown. The target heavy cold is shown. The sonic point is shown.
The ablation velocity is the speed at which the ablation front penetrates into the target. It can be calculated from the 1D theory results

\[ \dot{m}_a = 3.3 \cdot 10^5 \left( \frac{I_{15}}{\lambda_L^4} \right)^{1/3} \text{ g/cm}^2 \text{s} \]

\[ \dot{m}_a = \rho_{\text{shell}} u_a \quad u_a = \frac{\dot{m}_a}{\rho_{\text{shell}}} \]

\[ \rho_{\text{shell}} (\text{g/cc}) = \left( \frac{P(\text{Mbar})}{2.2\alpha} \right)^{3/5} \]

\[ P_a = 83 \left( \frac{I_{15}}{\lambda_L(\mu\text{m}) / 0.35} \right)^{2/3} \text{ Mbar} \]

\[ u_a = 1.1 \cdot 10^5 \frac{\alpha^{0.6}}{I_{15}^{0.067}} \left( \frac{0.35}{\lambda_L} \right)^{0.93} \text{ cm/s} \]
The ABLATIVE R-T is just Newton’s law at work again but with a restoring force: the dynamic pressure.

\[ \kappa S \left[ P_h - (P_\ell + \rho_\ell u_\ell^2) \right] = \rho_h S \ddot{\eta} \]

- **Newton’s law**
  - Energy balance
  \[ \frac{5}{2} \rho u_b = -q_{\text{heat}} \Rightarrow \ddot{u}_b = k u_b \ddot{\eta} \]
  - Perturbed dynamic pressure
  \[ \rho_\ell u_b \ddot{u}_b = k m u_b \ddot{\eta} \]

- Growth rate:
  \[ k S (\rho_h g - k m u_b) \ddot{\eta} = \rho_h S \dddot{\eta} \Rightarrow \dddot{\eta} \sim e^{\gamma t} \Rightarrow \gamma = \sqrt{kg - k^2 \frac{m}{\rho_h} u_b} \]
Proof required from previous slide

Start from flux balances

\[
\frac{5}{2} p u_b = -q_{\text{heat}} = \kappa \nabla T
\]

\[
u_b = u_b^0 + \tilde{u}_b
\]

\[\leftarrow \text{Linearize: equilibrium}^0 + \text{perturbation}^~\]

T = T^0 + \tilde{T}

Use equilibrium fluxes are \rightarrow balanced

\[
\frac{5}{2} p(u_b^0 + \tilde{u}_b) = \kappa \nabla T^0 + \kappa \nabla \tilde{T} \Rightarrow \frac{5}{2} p \tilde{u}_b = \kappa \nabla \tilde{T}
\]

\[
\frac{5}{2} \tilde{p} \tilde{u}_b = \kappa \kappa \tilde{T}
\]

find \[\nabla \sim k\]

Use

Define \rightarrow temperature perturbation

\[\tilde{T} \equiv T^0 (\tilde{\eta}) - T^0 \text{ since } T(\tilde{\eta}) = T^0 + \frac{d T^0}{dx} \tilde{\eta} \Rightarrow \tilde{T} = \frac{d T^0}{dx} \tilde{\eta}\]

\[
\frac{5}{2} p \tilde{u}_b = k k \frac{d T^0}{dx} \tilde{\eta} = \frac{5}{2} p u_b^0 k \tilde{\eta} \Rightarrow \tilde{u}_b = u_b^0 k \tilde{\eta}
\]

\[\leftarrow \text{Use equilibrium}^0 \]

\[\text{Taylor expansion}\]
Another stabilizing effect is the physical removal of the perturbation through ablation

\[ m_a = \rho_{\text{shell}} u_a \]
\[ \Delta x_a = u_a \Delta t \]
\[ \gamma_{\text{cl}} = \sqrt{kg} \]

- Classical: \( \tilde{v}(t, x) \sim e^{-k x + \gamma_{\text{cl}} t} \)
- Front frame (\( x' \)): \( x = x' + u_a t \)
- In the front frame: \( \tilde{v}(t, x') \sim e^{(\gamma_{\text{cl}} - ku_a)t - kx'} \) \( \rightarrow \gamma = \gamma_{\text{cl}} - ku_a \)
Another stabilizing effect is the ablation-driven convection of the vorticity off the ablation front.

\[ \gamma = \gamma_{cl} - ku_a \]
A cutoff in the unstable spectrum limits the number of unstable modes

- The full Ablative-RT growth rate includes all these effects:

\[ \gamma = \sqrt{k^2 - \frac{k^2}{\rho_h} m u_b + 4 k^2 u_a^2 - 2 ku_a} \]

- The cutoff wave number depends only on the dynamic pressure:

\[ kg = k^2 \frac{m}{\rho_h} u_b \Rightarrow k_{\text{cutoff}} = \frac{\rho_h g}{m u_b} \]

- Numerical fit (Takabe’s formula):

\[ \gamma \approx 0.9 \sqrt{kg - 3 ku_a} \]
The ablative growth is significantly less than the classical value. Modes with $k > k_c$ are stable.

\[ u_a = 3.5 \mu m/\text{ns} \]
\[ g = 100 \mu m/\text{ns}^2 \]

\[ k(\mu m^{-1}) = \frac{2\pi}{\lambda} \]
Only modes with $k\Delta \sim 1$ break the targets because the distortion inside the target decays in space $\eta(x) = \eta_f e^{-kx}$

- Rear surface distortion $\eta_r = \eta_f e^{-k\Delta}$
  - $k\Delta << 1$ does not break shell
  - $k\Delta >> 1$ does not break shell
  - $k\Delta \sim 1$ breaks shell!
Most dangerous modes have mode number equal to the In-Flight-Aspect-Ratio IFAR

- Wave number in planar geometry
  \[ k = \frac{2\pi}{\lambda} \]

- Wavelength in spherical geometry:
  \[ \lambda = \frac{2\pi R}{\ell} \]

- Wave number in spherical geometry
  \[ k = \frac{2\pi}{\lambda} = \frac{\ell}{R} \]

- Most dangerous modes in spherical geometry
  \[ k\Delta = \frac{\ell\Delta}{R} = \frac{\ell}{IFAR} = 1 \]

- Aspect ratio of the target studied in previous lecture IFAR\(\approx 70\)

- Most dangerous modes of our target
  \[ \ell \approx 70 \]
How much does a perturbation grow during the acceleration phase due to the (linear) RT instability?

\[ \eta(t) = \eta(0) e^{\gamma t} \]

\[ \gamma = 0.9 \sqrt{k g - 3k u_a} \]

\[ \gamma t = 0.9 \sqrt{kg t^2} - 3k u_a t = 0.9 \sqrt{(k \Delta) \frac{gt^2}{\Delta}} - 3(k \Delta) \frac{u_a t}{\Delta} \]

\[ gt^2 = R_0 \]

\[ t = R_0 / u_{\text{max}} \]

\[ \gamma t = 0.9 \sqrt{\frac{R_0}{\Delta}} - 3 \frac{u_a}{u_{\text{max}}} \frac{R_0}{\Delta} = 0.9 \sqrt{\text{IFAR}} - 3 \frac{u_a}{u_{\text{max}}} \cdot \text{IFAR} \]

Use \( u_a = 2.2 \times 10^5 \text{cm/s} \), \( u_{\text{max}} = 4.9 \times 10^7 \text{cm/s} \), IFAR=70

\[ \gamma t \approx 6.5 \quad \Rightarrow \quad \text{growth factor} = e^{\gamma t} = 665 \]
What if the initial perturbations on the targets are so large that the RT become immediately nonlinear (\(\rightarrow\) multimode interaction) and a turbulent mixing front develop?

- Drop mode wavelengths as scale lengths
- Only scale length left is \(gt^2\)
- Mixing front of width \(h\) advances according to
  \[
  h \sim \beta gt^2 = 2\beta \text{distance} \approx \beta R_1
  \]
- The figure of merit is the size of the mixing front to the target thickness
  \[
  \frac{h}{\Delta} \approx \beta \frac{R_1}{\Delta} \approx \beta \cdot \text{IFAR}
  \]
- RT simulations/experiments gives \(\beta \approx 0.05\)
- Our target with IFAR=70 would be fully mixed \(\rightarrow\) NO SHELL LEFT!

\[
\frac{h}{\Delta} \approx 0.05 \cdot 70 = 3.5 > 1
\]

MUST CONTROL THE SEEDS OF THE RT \(\rightarrow\) MAKE SMOOTH TARGETS AND SMOOTH LASER BEAMS
Lot of work on hydrodynamic instabilities needs to be done

- Multimode, turbulent Rayleigh-Taylor instability is not well understood

- The effect of ablation on the nonlinear multimode evolution is not well understood

- The effect of the initial conditions on the turbulent front dynamics is not well understood

- This is important stuff for inertial fusion!