

LECTURE # 3

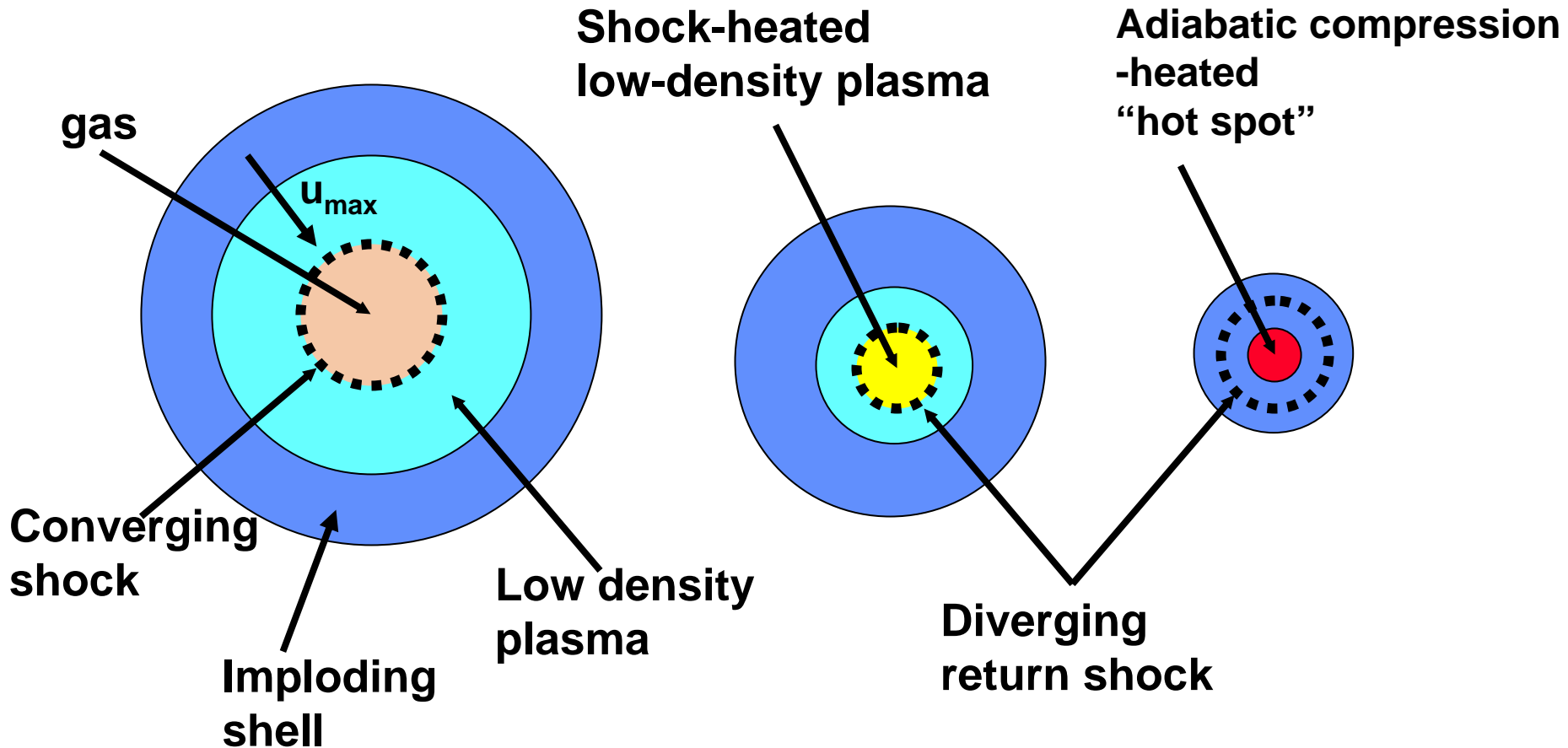
HOT SPOT DYNAMICS AND HYDRODYNAMIC INSTABILITIES

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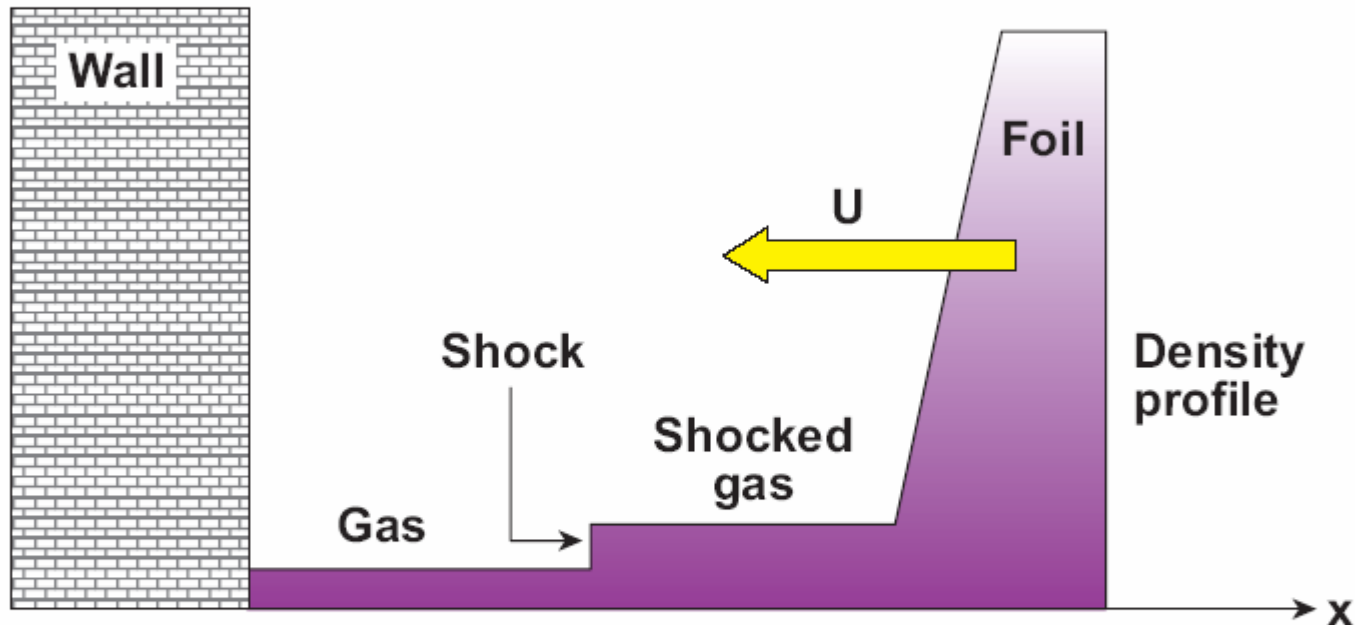
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Hot spot dynamics in 1D spherical implosions



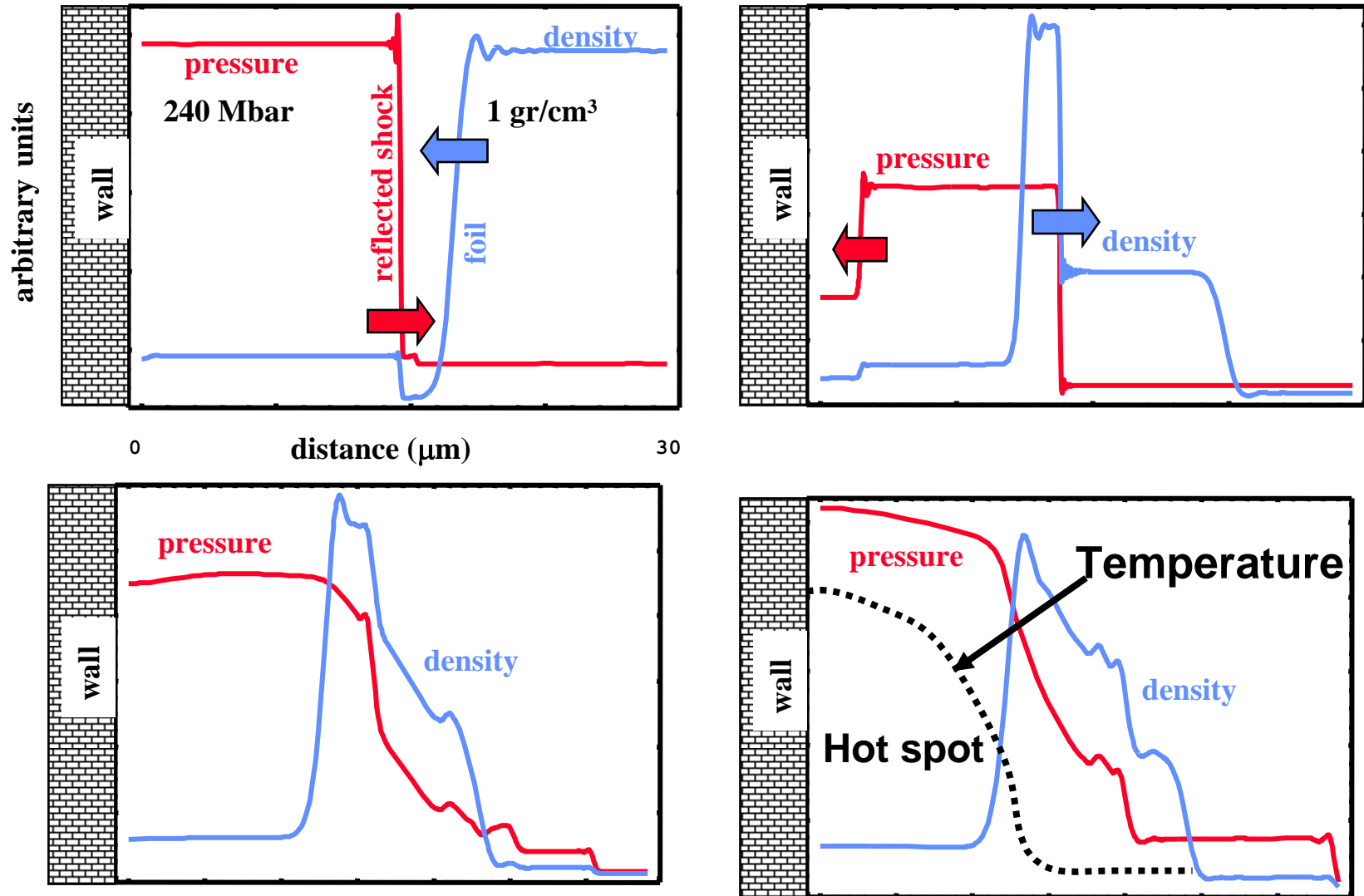
The decelerating slab problem provides the basic understanding of the deceleration phase and hot spot formation



- The shock reflected from the wall slows down the foil, which in turn compresses the gas and decelerates.
- The 1-D problem can be solved analytically leading to a clear understanding of the relevant physics issues.

The low density gas is heated to form a hot spot

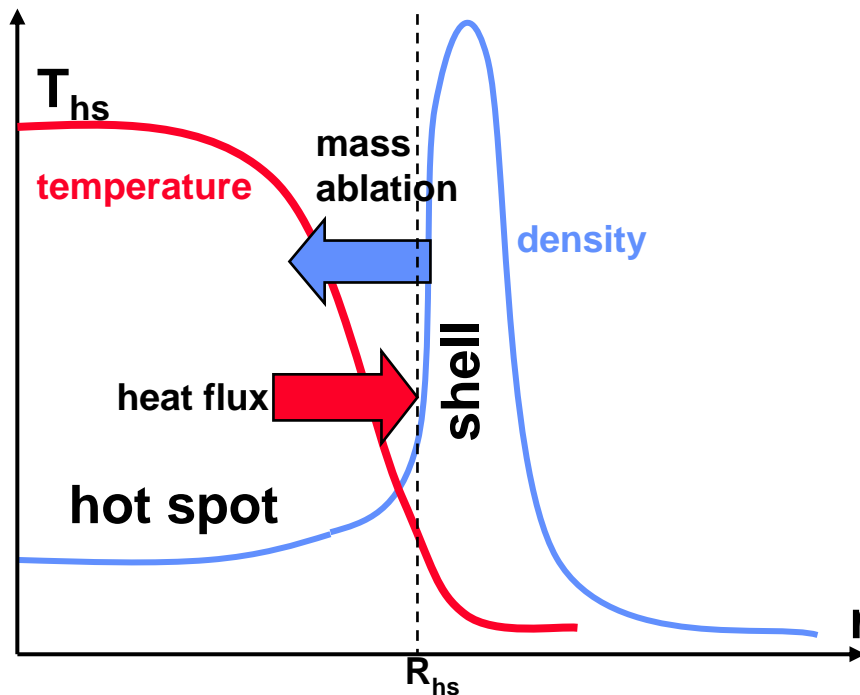
Evolution of pressure (red) and density (blue).



The heat flux leaving the hot spot is deposited onto the shell surface causing mass ablation from the shell into the hot spot. The hot spot mass increases in time.

$$q_{heat} = -\kappa(T)\nabla T$$

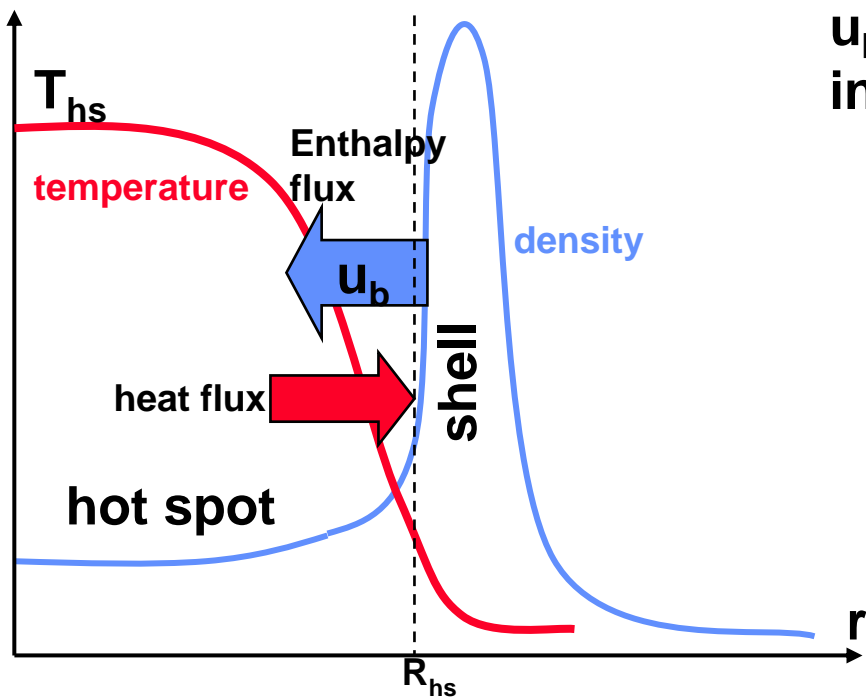
$$\kappa(T) \approx \kappa_0 T^{5/2}$$



The heat leaving the hot spot cannot penetrate the shell because the shell is cold and its thermal conductivity is low,

$$\kappa_{shell} \ll \kappa_{hot\ spot}$$

The heat is deposited on the shell inner surface causing mass ablation off the shell



$u_b =$ blow-off velocity
into hot spot

$$\frac{5}{2} p u_b = \left[-\kappa_0 T^{5/2} \frac{dT}{dR_{hs}} \right]_{R_{hs}^-}$$

Enthalpy flux
Heat flux

- Hot spot temperature profile $\rightarrow T_{hs} = T_0 \left(1 - \frac{r^2}{R_{hs}^2} \right)^{2/5}$
 - Use ideal gas EOS: $p u_b = 2 \rho_{R_{hs}} T_{R_{hs}} u_b / m_i = 2 \dot{m} T_{R_{hs}} / m_i$
 - Ablation rate into hot spot: $\dot{m} = \rho_{R_{hs}} u_b$
- substitute
find
- $$\dot{m} = 0.2 \frac{m_i \kappa_0 T_0^{5/2}}{R_{hs}}$$

Hot spot volume $\sim R_{hs}^3$

Hot spot density

Use EOS $\rho = m_i p / 2T$

Hot spot
Mass \rightarrow

$$M_{hs} = \rho_{hs} V_{hs} = \frac{m_i}{2} \frac{p V_{hs}}{T_{hs}}$$

Mass
conservation \rightarrow

$$\frac{dM_{hs}}{dt} = 4\pi R_{hs}^2 \dot{m} \quad \rightarrow \quad m_i \frac{p V_{hs}}{t T} \sim R_{hs}^2 \frac{m_i \kappa_0 T^{5/2}}{R_{hs}}$$

Hot spot
compression time \rightarrow

$$t \sim \frac{R_{hs}}{u_{max}} \quad p V_{hs} \sim M_{shell} u_{max}^2$$

\leftarrow **Energy Conservation**
(hot spot internal energy comes from shell kinetic energy)

Hot spot \rightarrow
temperature

$$T \sim \left(\frac{M_{shell} u_{max}^3}{\kappa_0 R_{hs}^2} \right)^{2/7} \sim \frac{1}{\kappa_0^{2/7}} \left[(\rho R)_{shell}^{stag} \right]^{2/7} u_{max}^{6/7}$$

\leftarrow **Shell areal density**

- Use the areal density scaling found in previous lecture

$$\left(\rho R\right)_{stag}^{shell} \sim \frac{E_L^{1/3} u_{\max}^{2/3} I^{4/45}}{\alpha^{4/5}}$$

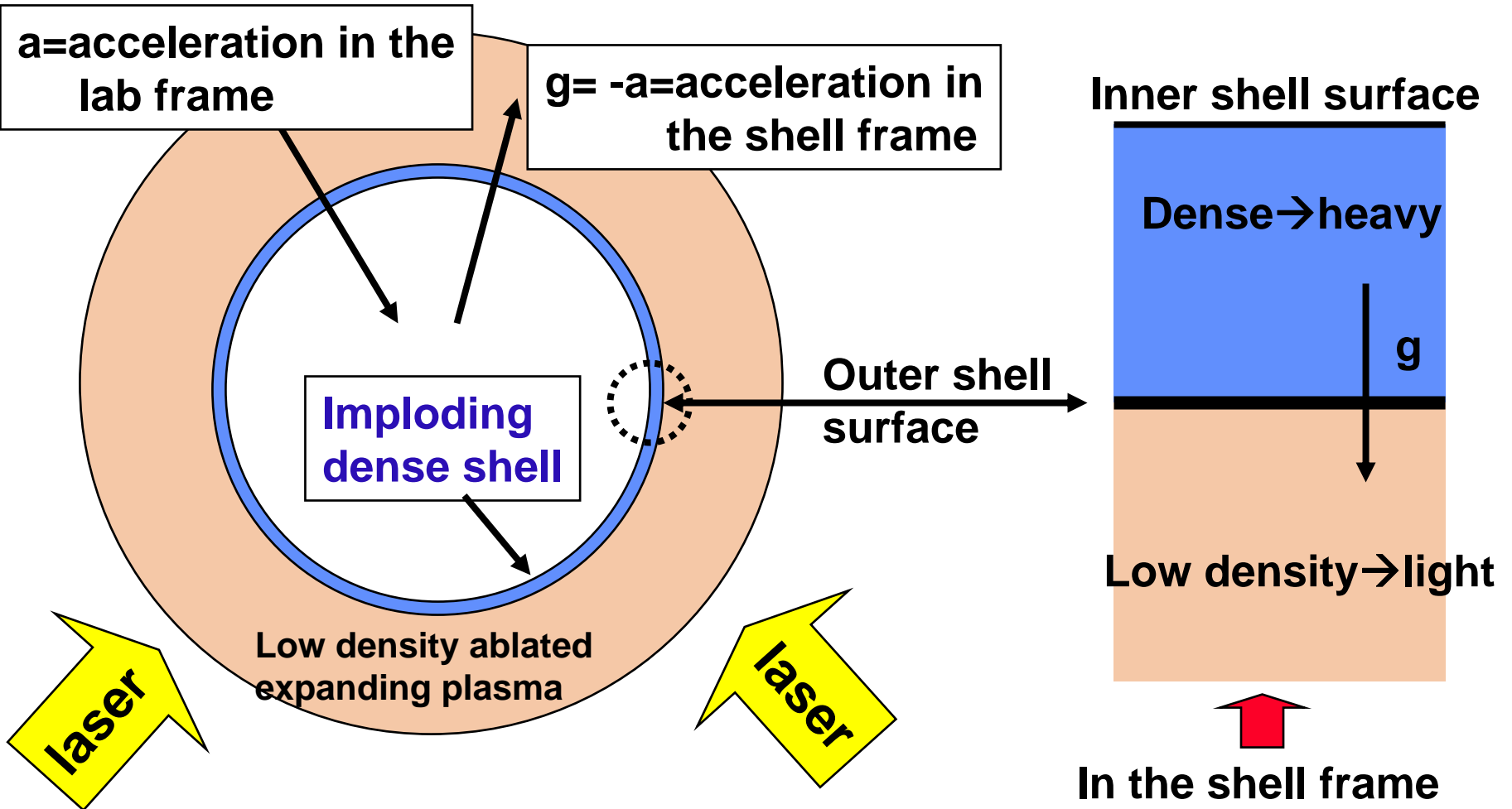
- Find the temperature scaling. The hot spot temperature mainly depends on the implosion velocity.

$$T_{hot-spot} \sim \frac{E_L^{0.1} I^{0.03}}{\alpha^{0.2}} u_{\max}^{1.1}$$

- Simulation results (without α -heating) confirm the theory

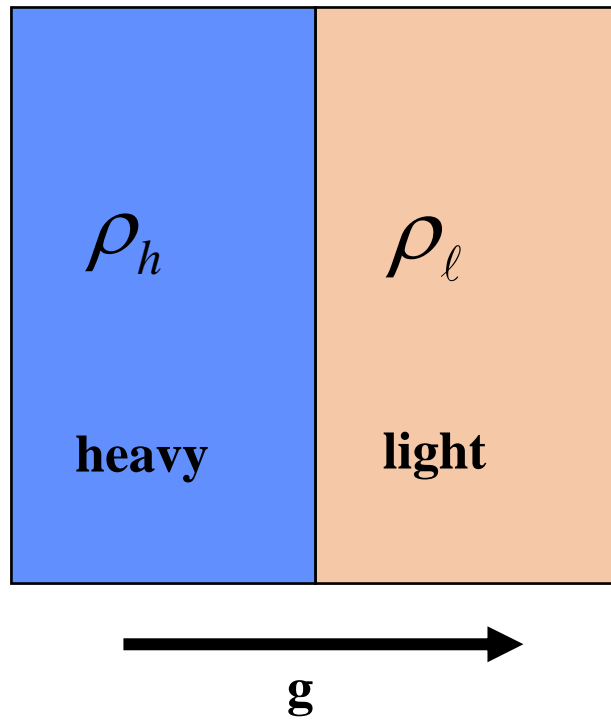
$$\langle T_{hot-spot} \rangle (keV) \approx \frac{3}{\alpha^{0.15}} \left(\frac{u_{\max} (cm/s)}{3 \times 10^7} \right)^{1.25} \left(\frac{E_L (kJ)}{100} \right)^{0.07}$$

HYDRODYNAMIC INSTABILITIES

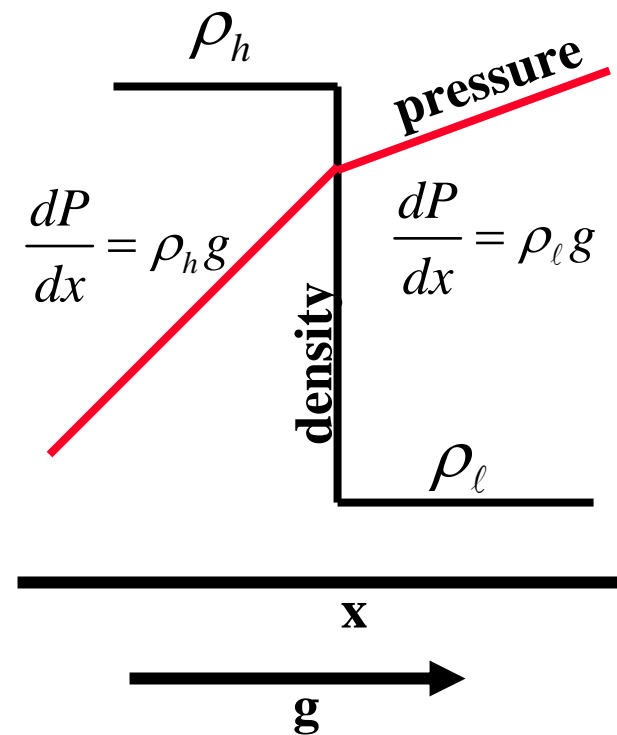


THE CLASSICAL RAYLEIGH-TAYLOR INSTABILITY of A HEAVY FLUID SUPPORTED BY A LIGHTER FLUID

EQUILIBRIUM CONDITIONS

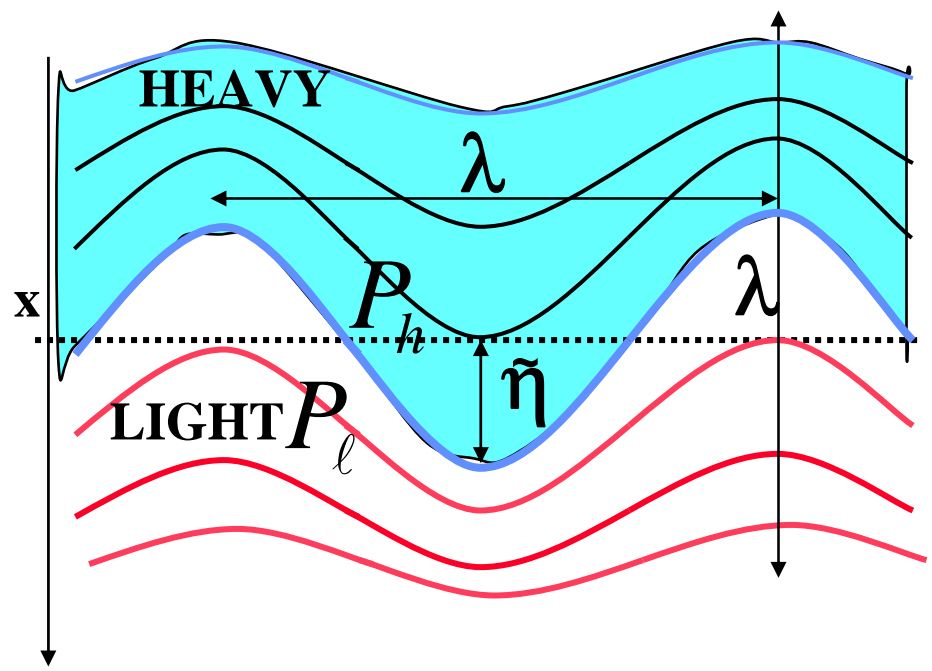


Pressure gradient is
opposite to density gradient



Acceleration Phase

The classical R-T is just Newton's law at work: $F=ma!$



$$k \sim 1/\lambda$$

$$F = S(P_h - P_l) = ma = \rho_h \lambda S \ddot{\tilde{\eta}}$$

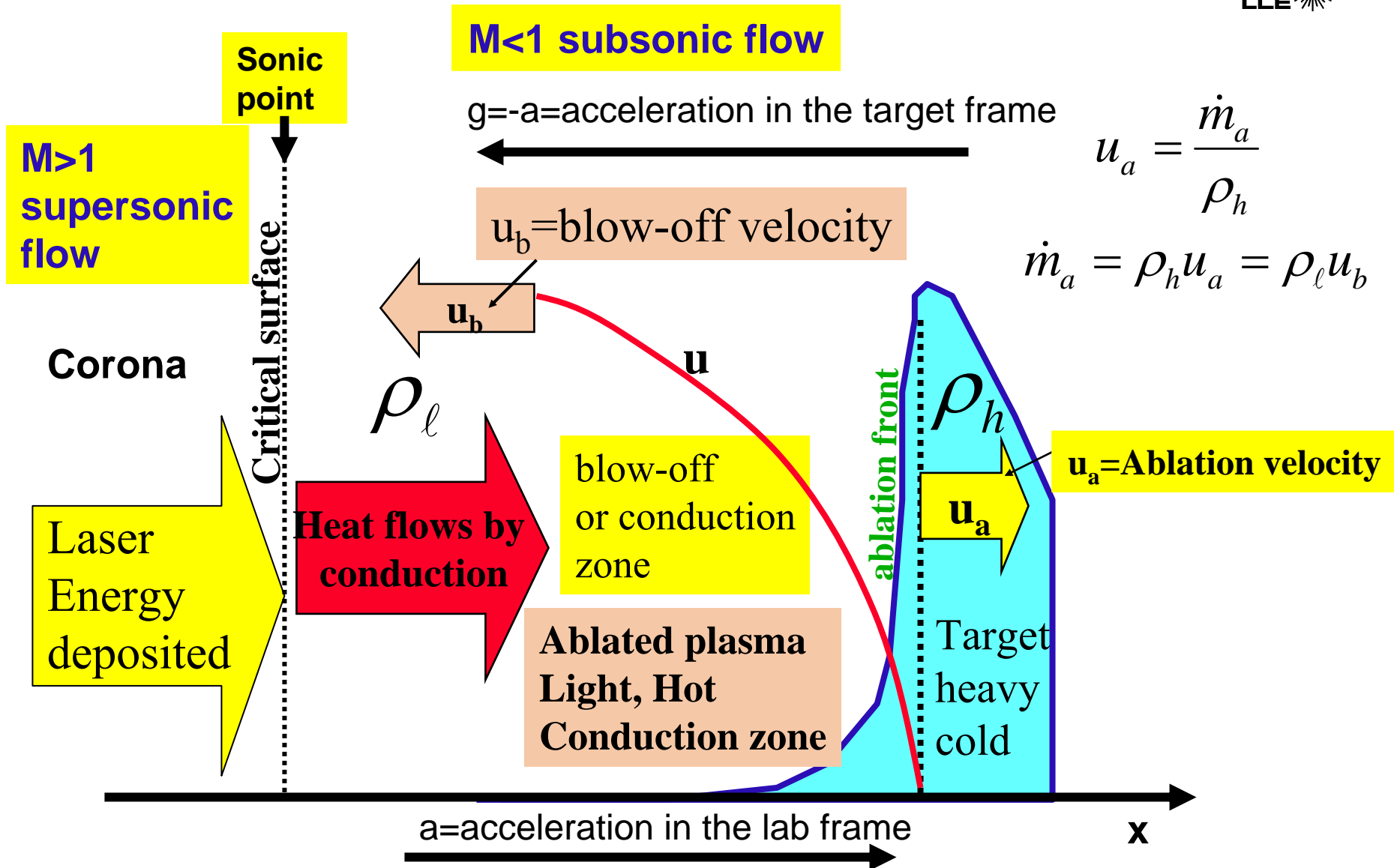
$$\frac{dP_0}{dx} = \rho_0 g = \begin{cases} \rho_h g & \text{heavy} \\ \rho_l g & \text{light} \end{cases}$$

$$P_l = P_0 + \left[\frac{dP_0}{dx} \right]_l \tilde{\eta} = P_0 + \cancel{\rho_l g} \tilde{\eta}$$

$$P_h = P_0 + \left[\frac{dP_0}{dx} \right]_h \tilde{\eta} = P_0 + \rho_h g \tilde{\eta}$$

$$F = ma \rightarrow S \rho_h g \tilde{\eta} = \rho_h \lambda S \ddot{\tilde{\eta}} \rightarrow \ddot{\tilde{\eta}} = kg \tilde{\eta} \rightarrow \tilde{\eta} \sim e^{\gamma t} \rightarrow \gamma = \sqrt{kg}$$

In the laser irradiated targets the heat/ablation front penetrates at the ablation velocity



The ablation velocity is the speed at which the ablation front penetrates into the target. It can be calculated from the 1D theory results

$$\dot{m}_a = 3.3 \cdot 10^5 (I_{15} / \lambda_L^4)^{1/3} \text{ g / cm}^2 \text{ s}$$

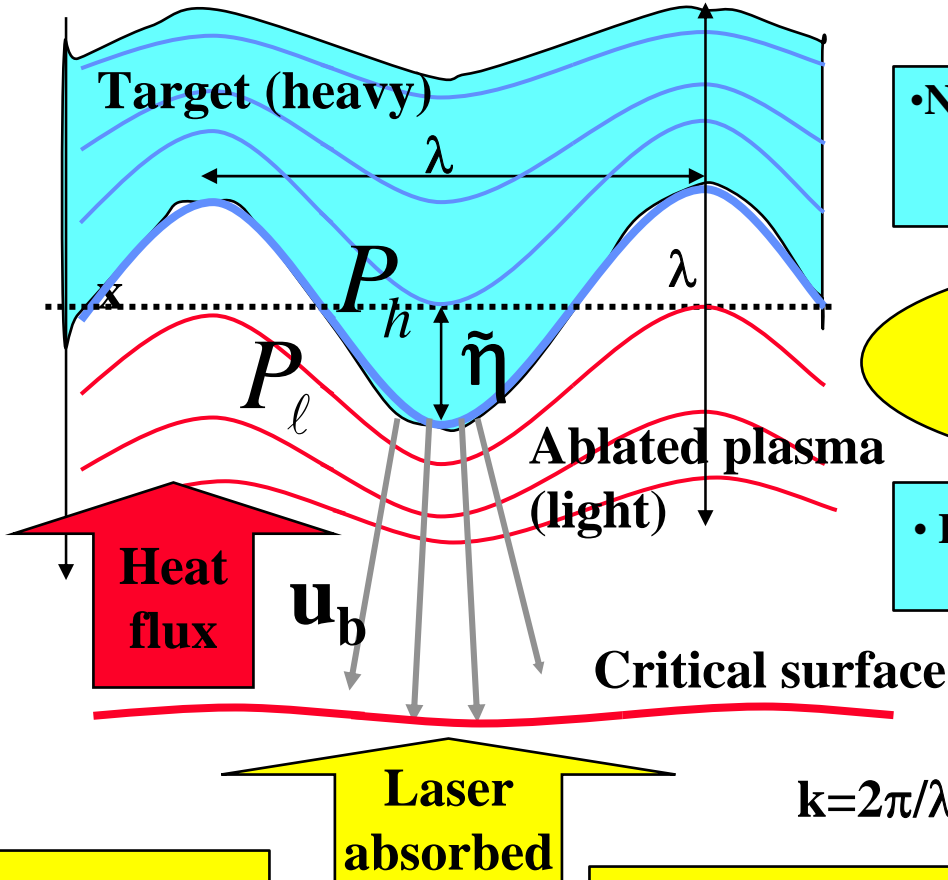
$$\dot{m}_a = \rho_{shell} u_a \quad u_a = \dot{m}_a / \rho_{shell}$$

$$\rho_{shell} (\text{g / cc}) = \left(\frac{P(\text{Mbar})}{2.2\alpha} \right)^{3/5}$$

$$P_a = 83 \left(\frac{I_{15}}{\lambda_{L(\mu m)} / 0.35} \right)^{2/3} \text{ Mbar}$$

$$u_a = 1.1 \cdot 10^5 \frac{\alpha^{0.6}}{I_{15}^{0.067}} \left(\frac{0.35}{\lambda_L} \right)^{0.93} \text{ cm / s}$$

The ABLATIVE R-T is just Newton's law at work again but with a restoring force: the dynamic pressure.



• Newton's law

$$kS [P_h - (P_l + \rho_l u_b^2)] = \rho_h S \ddot{\tilde{\eta}}$$

• Energy balance

$$\frac{5}{2} \rho u_b = -q_{heat} \Rightarrow \tilde{u}_b = k u_b \tilde{\eta}$$

• Perturbed dynamic pressure

$$\underbrace{\rho_l u_b}_{\text{Ablation rate}} \tilde{u}_b = k m u_b \dot{\tilde{\eta}}$$

Need to show

stabilizing

• Growth rate: $kS(\rho_h g - k \dot{m} u_b) \tilde{\eta} = \rho_h S \ddot{\tilde{\eta}} \rightarrow \tilde{\eta} \sim e^{\gamma t} \rightarrow \gamma = \sqrt{kg - k^2 \frac{\dot{m}}{\rho_h} u_b}$

Restoring force

Proof required from previous slide

Start from flux balances

$$\frac{5}{2} p u_b = -q_{heat} = \kappa \nabla T$$

$$u_b = u_b^0 + \tilde{u}_b$$

← Linearize:
equilibrium ⁰
+
perturbation \sim

$$T = T^0 + \tilde{T}$$

Use equilibrium
fluxes are →
balanced

$$\frac{5}{2} p (u_b^0 + \tilde{u}_b) = \cancel{\kappa \nabla T^0} + \kappa \nabla \tilde{T} \Rightarrow \frac{5}{2} p \tilde{u}_b = \kappa \nabla \tilde{T}$$

$$\frac{5}{2} p \tilde{u}_b = \cancel{\kappa} \tilde{T} \quad \xleftarrow{\text{find}} \quad \tilde{\nabla} \sim \cancel{\kappa} \quad \xleftarrow{\text{Use}}$$

Define →
temperature
perturbation

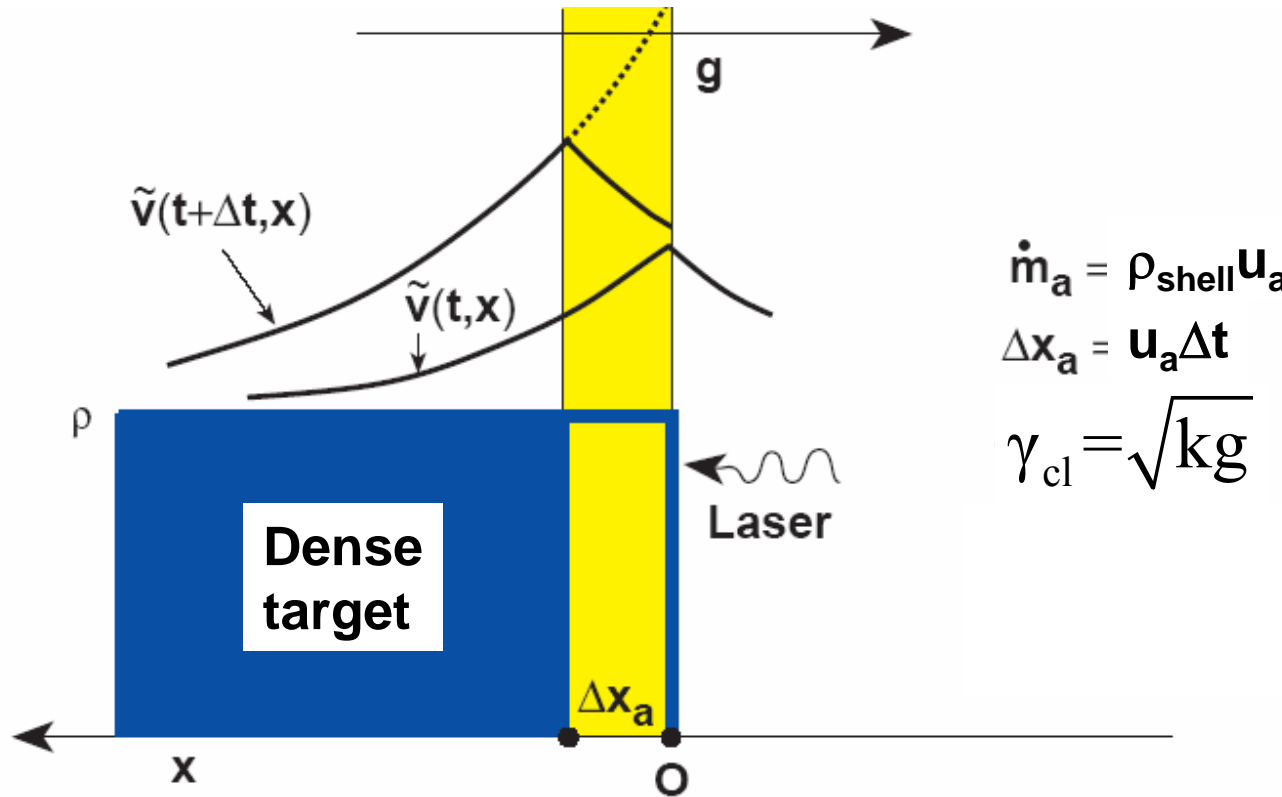
$$\tilde{T} \equiv T^0(\tilde{\eta}) - T^0 \quad \text{since} \quad T(\tilde{\eta}) = T^0 + \frac{dT^0}{dx} \tilde{\eta} \Rightarrow \tilde{T} = \frac{dT^0}{dx} \tilde{\eta}$$

$$\frac{5}{2} p \tilde{u}_b = \cancel{\kappa} \frac{dT^0}{dx} \tilde{\eta} = \frac{5}{2} p u_b^0 \cancel{\kappa} \tilde{\eta} \Rightarrow \tilde{u}_b = u_b^0 \tilde{\eta}$$

Taylor
expansion

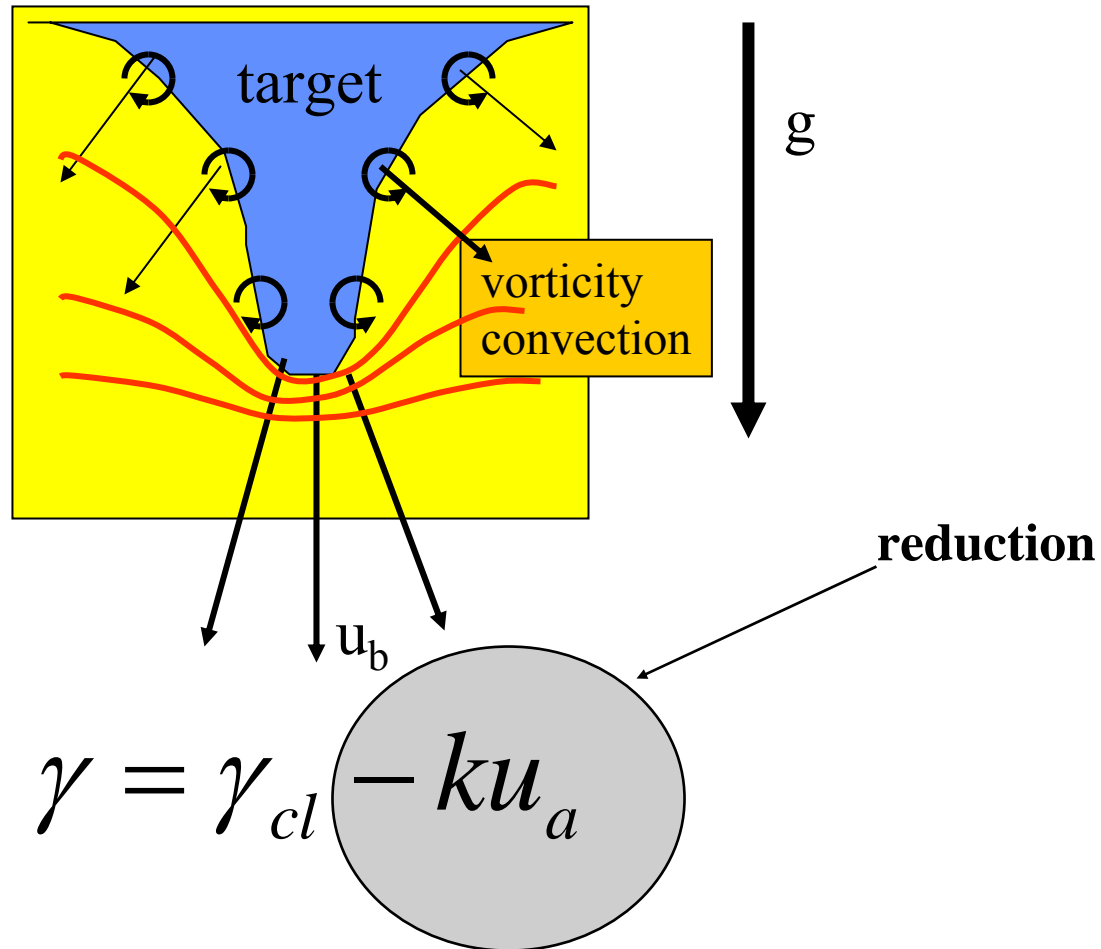
← Use equilibrium

Another stabilizing effect is the physical removal of the perturbation through ablation



- Classical: $\tilde{v}(t, x) \sim e^{-kx + \gamma_{\text{cl}} t}$
- Front frame (x'): $x = x' + \mathbf{u}_a t$
- In the front frame: $\tilde{v}(t, x') \sim e^{(\gamma_{\text{cl}} - k\mathbf{u}_a)t - kx'}$ $\rightarrow \gamma = \gamma_{\text{cl}} - k\mathbf{u}_a$

Another stabilizing effect is the ablation-driven convection of the vorticity off the ablation front



A cutoff in the unstable spectrum limits the number of unstable modes

- The full Ablative-RT growth rate includes all these effects:

$$\gamma = \sqrt{kg - k^2 \frac{\dot{m}}{\rho_h} u_b + 4k^2 u_a^2 - 2ku_a}$$

Annotations:

- dynamic pressure (points to kg)
- Mass removal + vorticity convection (points to $-2ku_a$)
- Both of these effects cannot fully suppress the instability (points to the entire expression)

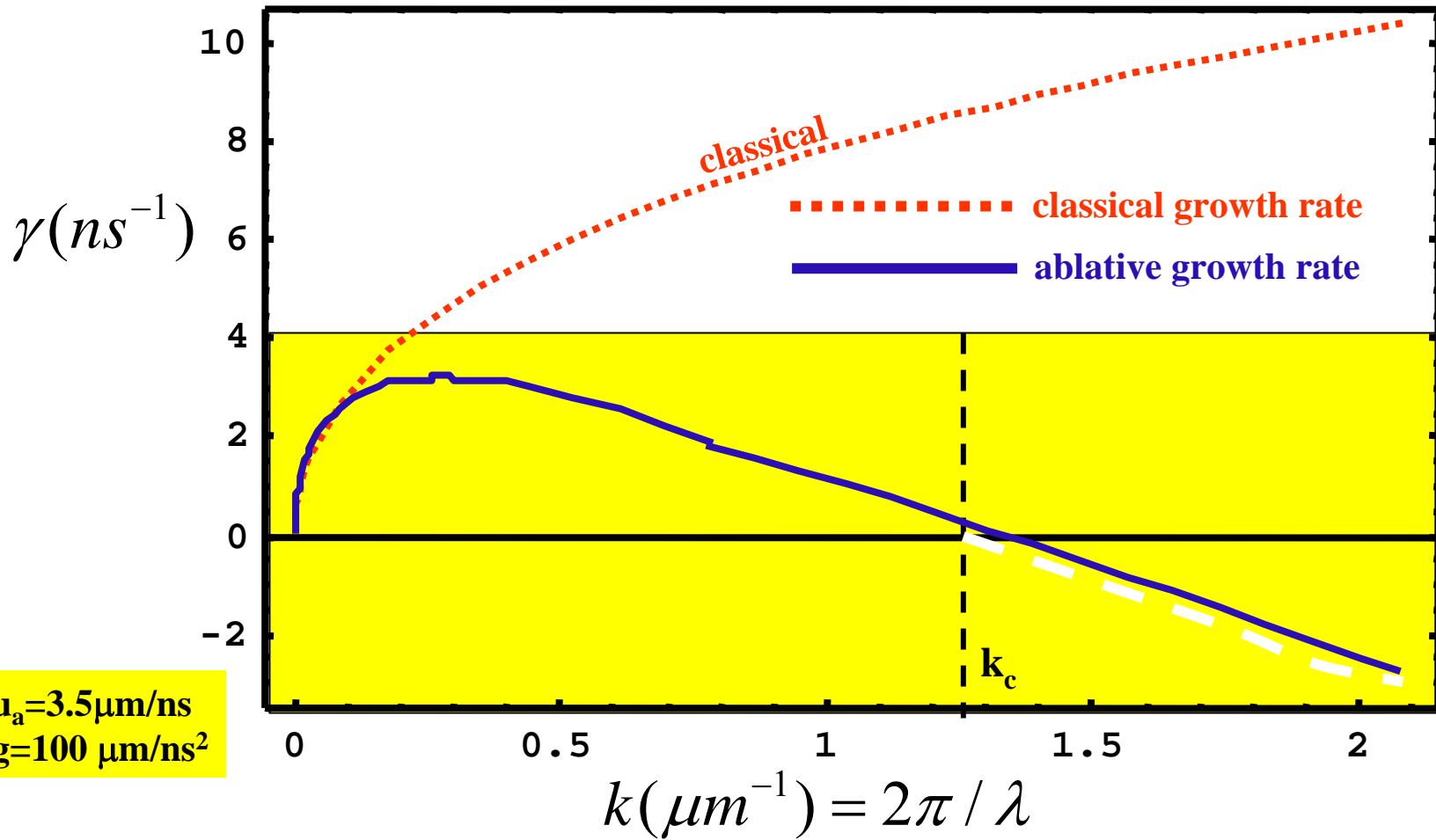
- The cutoff wave number depends only on the dynamic pressure:

$$kg = k^2 \frac{\dot{m}}{\rho_h} u_b \quad \rightarrow \quad k_{cutoff} = \frac{\rho_h g}{\dot{m} u_b} \quad \leftarrow \quad u_b = u_a \frac{\rho_h}{\rho_l}$$

- Numerical fit (Takabe's formula):

$$\gamma \approx 0.9 \sqrt{kg - 3ku_a}$$

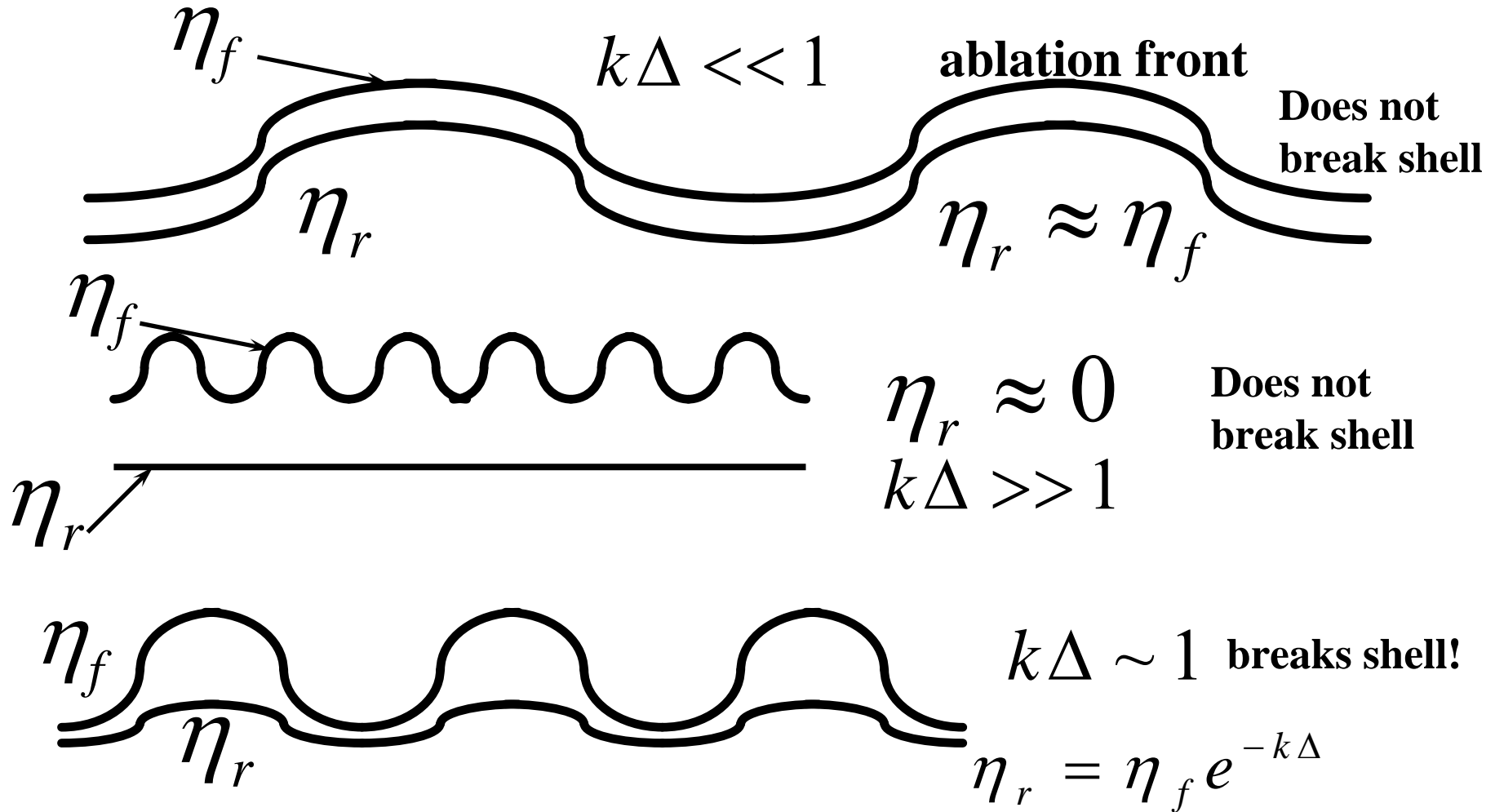
The ablative growth is significantly less than the classical value. Modes with $k > k_c$ are stable



$u_a = 3.5 \mu m/ns$
 $g = 100 \mu m/ns^2$

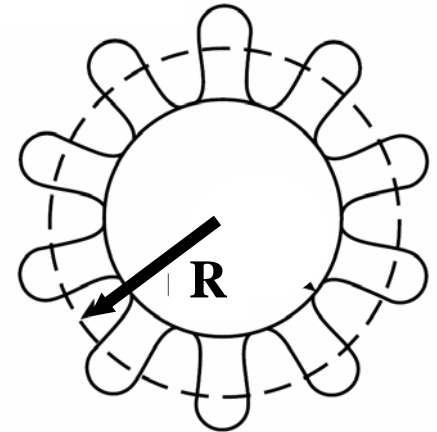
Only modes with $k\Delta \sim 1$ break the targets because the distortion inside the target decays in space $\eta(x) = \eta_f e^{-kx}$

•Rear surface distortion $\rightarrow \eta_r = \eta_f e^{-k\Delta}$



Most dangerous modes have mode number equal to the In-Flight-Aspect-Ratio IFAR

- Wave number in planar geometry $k = 2\pi / \lambda$
- Wavelength in spherical geometry: $\lambda = 2\pi R / \ell$
- Wave number in spherical geometry $k = 2\pi / \lambda = \ell / R$
- Most dangerous modes in spherical geometry



$$k\Delta = \ell\Delta / R = \ell / IFAR = 1 \quad \longrightarrow \quad \ell_{\text{most-dangerous}} = IFAR$$

- Aspect ratio of the target studied in previous lecture $IFAR \sim 70$

- Most dangerous modes of our target $\rightarrow \ell \sim 70$

How much does a perturbation grow during the acceleration phase due to the (linear) RT instability?

$$\eta(t) = \eta(0) e^{\gamma t} \quad \gamma = 0.9\sqrt{kg} - 3ku_a$$

$$\gamma t = 0.9\sqrt{kg}t^2 - 3ku_a t = 0.9\sqrt{(k\Delta)\frac{gt^2}{\Delta}} - 3(k\Delta)\frac{u_a t}{\Delta}$$

$$gt^2 = R_0 \quad t = R_0 / u_{\max}$$

Take $k\Delta=1$,
most dangerous
modes

$$\gamma t = 0.9\sqrt{\frac{R_0}{\Delta}} - 3\frac{u_a}{u_{\max}}\frac{R_0}{\Delta} = 0.9\sqrt{IFAR} - 3\frac{u_a}{u_{\max}}IFAR$$

Use $u_a=2.2\cdot 10^5$ cm/s, $u_{\max}=4.9\cdot 10^7$ cm/s, $IFAR=70$

$$\gamma t \approx 6.5 \quad \Rightarrow \quad \text{growth factor} = e^{\gamma t} = 665$$

What if the initial perturbations on the targets are so large that the RT become immediately nonlinear (\rightarrow multimode interaction) and a turbulent mixing front develop?

- Drop mode wavelengths as scale lengths
- Only scale length left is gt^2
- Mixing front of width h advances according to

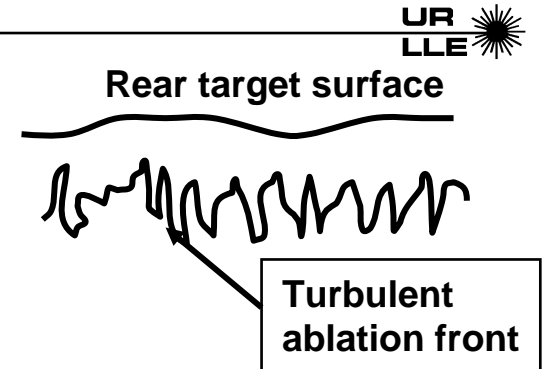
$$h \sim \beta gt^2 = 2\beta \text{distance} \approx \beta R_1$$

- The figure of merit is the size of the mixing front to the target thickness

$$\frac{h}{\Delta} \approx \beta \frac{R_1}{\Delta} \approx \beta \bullet IFAR$$

- RT simulations/experiments gives $\beta \approx 0.05$
- Our target with $IFAR=70$ would be fully mixed \rightarrow NO SHELL LEFT!

$$\frac{h}{\Delta} \approx 0.05 \bullet 70 = 3.5 > 1$$



MUST CONTROL THE SEEDS OF THE RT \rightarrow MAKE SMOOTH TARGETS AND SMOOTH LASER BEAMS

Lot of work on hydrodynamic instabilities needs to be done

- **Multimode, turbulent Rayleigh-Taylor instability is not well understood**
- **The effect of ablation on the nonlinear multimode evolution is not well understood**
- **The effect of the initial conditions on the turbulent front dynamics is not well understood**
- **This is important stuff for inertial fusion!**