What is nuclear fusion?

If there are sufficient reactions and the neutron energy is converted into heat, then this could be a formidable power source. Fusion power the stars and the H-bombs.

Deuterium + Tritium → Helium (alpha particle) + Neutron

3.5 MeV + 14 MeV
A “hot plasma” is needed to sustain many fusion reactions

-> Probability for fusion reaction to occur is low at low temperatures because of Coulomb repulsion force.

-> If the ions are sufficiently hot (e.g... large random velocity/kinetic energy) then they can collide by overcoming Coulomb repulsion.
Thermonuclear ignition of DT fuel occurs when the fusion reactions are maintained exclusively by the alpha heating.

The alpha particle is slowed down by collisions with the background plasma electrons. Its kinetic energy is transferred to the plasma → alpha heating → self-heating.

*Ignition occurs when the fusion reactions are maintained exclusively by the alpha heating.*
Fusion burn

\[ \frac{dn}{dt} = N_D N_T \langle \sigma_{DT} V \rangle \]

\[ \xi = \frac{2n}{N_0} \]

\[ \xi_0 = \frac{N_0 \tau_c}{1 - \xi_0} \frac{\langle \sigma_{DT} V \rangle}{2} \]

\[ \xi_0 = \frac{N_0 \tau_c \langle \sigma_{DT} V \rangle}{2 + N_0 \tau_c \langle \sigma_{DT} V \rangle} \]

\[ N_D = N_T = \left( \frac{1}{2} N_0 - n \right) \]

Fusion reaction rate is a function of temperature

\[ \xi_0 = \text{burned fraction} \quad \langle \sigma_{DT} V \rangle = \langle \sigma_{DT} V \rangle(T) \quad \tau_c = \text{burn time} \]
Temperatures of several keVs are needed for fusion.
Necessary (but not sufficient) condition for igniting DT fuel and minimum temperature of the fuel

\[ \frac{1}{4} \left( N^2 \langle \sigma v \rangle \varepsilon_\alpha \right) > C_b \left( N^2 \sqrt{T} \right) \]

\[ \frac{\langle \sigma v \rangle (T)}{\sqrt{T}} > \text{constant} \]

\[ T > 4.4 \text{keV} \]
An isolated burning sphere of thermonuclear fuel would lose thermal energy mostly by hydrodynamic expansion.

An ignited plasma keeps burning as long as

\[ P_\alpha > P_{\text{rad}} + P_{\text{expansion}} = P_{\text{losses}} = \frac{W_{\text{plasma}}}{\tau_c} \]

Since an ignited plasma has \( T \sim 20-40\text{keV} \), radiation losses are small \( \rightarrow \) confinement time = expansion time = burn time.
An isolated hot burning sphere of DT fuel expands outward at about 3 times the sound speed. The fraction burned depends on the areal density $\rho R$.

- Hydrodynamic confinement time: $\tau_c \sim \frac{R}{3C_s}$

- The sound speed depends on temperature: $C_s \sim \sqrt{\frac{T}{m_i}}$

\[
\xi_0 = \frac{N_0 R}{N_0 R + 6C_s / \langle \sigma_{DT} V \rangle}
\]

\[
\xi_0 = \frac{\rho R}{\rho R + 7g/cm^2}
\]

A $\rho R = 3g/cm^2$ is needed for a 30% burn-up.
Can one make a sphere of cryogenic DT fuel with \( \rho R = 3 \, \text{g/cm}^2 \) and ignite the whole thing?

- The DT solid density is 0.25g/cc
- To make a \( \rho R = 3 \) sphere one needs R~12cm and 1.8kg of DT
- If ignited, it would produce 190 Terajoules
- A kiloton of TNT is equivalent to 4.2 Terajoules
- Our sphere is equivalent to 45 kilotons
- The Hiroshima bomb had a yield of 13 kilotons
- Our sphere is equivalent to 3.5 Hiroshimas
- The confinement time for R=12cm at 5keV is about 100ns
- It takes 1TJ of energy to heat up the sphere at 5keV for ignition
- This heat must be provided on the confinement time scale (100ns)
- **The power required is** \( \frac{1\,\text{TJ}}{100\,\text{ns}} = 10^7\,\text{TW} \) !!!
- The US electrical grid is only 3 TW!
Let’s start again. Can one ignite only a small portion and hope for a propagating burn just like when igniting gasoline with a spark plug?

• A necessary condition for burn propagation is that the alpha particles deposit their energy near where they are born

• The alpha particle range in a 10keV plasma is 0.3g/cm²

• The minimum spark size must be a sphere with \( R_s = 1.2\text{cm} \) (for solid DT)
• Minimum energy required to ignite the spark is 2GJ
• Spark confinement time (only hydro) is 6ns
• Power required to ignite is \( 3 \times 10^5 \text{ TW} \)

Still too much power and energy!
If lasers are used as power source then:
Size ~ hundreds microns – millimeters
Mass ~ milligrams
Densities ~ hundreds g/cc
Energies ~ Megajoules
Power ~ hundreds of TW

- Lasers deliver energy over nanosecond time scales \( t_L \sim 10^{-9} \text{ s} \)
- Equating laser time and hydro time yields the size
  \[ t_L \sim \frac{R}{C_s (T = 5 \text{ keV})} \Rightarrow R \sim 100 \mu m - 1 \text{ mm} \]
- The size and the required areal density yield the density
  \[ \rho R \sim 1 - 3 g / \text{ cm}^2 \Rightarrow \rho \sim 100 g / \text{ cc} \]
- The mass comes from size and density
  \[ M = (4 \pi / 3) \rho R^3 \sim 1 - 10 \text{ milligrams} \]
- Energy to heat 1mg DT to 5keV is 0.5MJ and power is 500TW

Not quite right. Better derivation shown later
A hot low-density low-mass DT plasma surrounded by a cold dense and massive DT shell seems to be the most efficient configuration to achieve “easy” ignition, propagating burn and large energy gains (energy out $>>$ energy in).

Goal: ignition starts in hot spot and burn wave propagate to the main fuel.
Ignition of the hot spot: the hot spot cannot freely expand because of the surrounding shell. The hydro time is much longer than the one of the freely expanding hot plasma.

- **Shell Newton’s law:** $M_{sh} \ddot{R} = 4\pi R^2 P$

- The hydro time of the hot spot is the hydro time of the shell

\[
\tau_h \sim \sqrt{\frac{R}{\ddot{R}}} \sim \sqrt{\frac{M_{sh}}{4\pi PR}}
\]

\[
M_{sh} \sim 4\pi \rho_{sh} R^2 \Delta
\]

\[
\tau_h \sim \sqrt{\frac{R\Delta}{P/\rho_{sh}}} \sim \frac{R}{C_s(T_{shell})} \gg \frac{R}{C_s(T_{hot-spot})}
\]
Another advantage of the cold dense shell is the thermal energy recycling occurring at the hot spot-shell interface.

- Hot plasmas lose energy by thermal conduction:
  \[ q_{\text{heat}} = -\kappa \nabla T \]
- Thermal conduction is only large if the temperature is high:
  \[ \kappa \approx \kappa_0 T^{5/2} \]
- The heat flux leaving the hot spot cannot go through the shell since \( \kappa \approx 0 \) in the shell.

The heat flux leaving the hot spot is deposited onto the shell surface causing mass ablation from the shell into the hot spot. The hot spot mass increases in time. The mass ablated takes the energy back into the hot spot.
The ablation rate from the shell to the hot spot is determined from the energy balance at the hot spot-shell interface.

\[
\left( \frac{3}{2} p_b V_b + p_b V_b \right) = \kappa_{\text{Spitz}}(T_b) \frac{dT_b}{dR_{hs}}
\]

Internal energy flux \hspace{1cm} pdV work rate

Energy flux balance

- Hot spot temperature profile: 
  \[ T_{hs} = T_0 \left( 1 - \frac{r^2}{R_{hs}^2} \right)^{2/5} \]

- Use the ideal gas EOS: 
  \[ p_b V_b = 2 \rho_b V_b T_b / m_i = 2 m T_b / m_i \]

- Ablation rate from the shell into the hot spot: 
  \[ m = 0.2 \frac{m_i \kappa_{\text{Spitz}}(T_0)}{R_{hot-spot}} \]
While the total energy inside the hot spot is not affected by the heat conduction losses, the hot spot temperature is.

- Heat conduction losses are recycled into the hot spot by ablation off the shell. The total energy density ~ P~nT is not affected.

- Heat conduction causes T decrease while ablation causes n to increase in such a way that P~nT remains unchanged.
Temperature or Pressure THIS IS THE QUESTION!

- In order to ignite the hot spot the alpha heating must take over

- Back to the power density deposited by the alphas (assume $\rho R \geq 0.3 \text{g/cm}^2$)

$$\dot{W}_\alpha = N_D N_T <\sigma v> \varepsilon_\alpha = \frac{1}{4} (NT)^2 \frac{<\sigma v>}{T^2} \varepsilon_\alpha = \frac{\varepsilon_\alpha P^2}{16} \frac{<\sigma v>}{T^2}$$

- For $27>T>6\text{keV}$, $\frac{<\sigma v>}{T^2} \sim \text{constant}$

then the alpha power depends only on pressure

$$\dot{W}_\alpha \approx \frac{\varepsilon_\alpha}{16} \sum_\alpha P^2$$

However, for lower T temperature does matter and

$$\dot{W}_\alpha \approx P^2 T \text{ or } P^2 T^2$$

$\sum_\alpha = 9.65 \times 10^{-19} \text{cm}^3/\text{s keV}^2$
Simplest ignition model:
Assume (1) $T>6\text{keV}$, (2) $\rho R \geq 0.3 \text{g/cm}^2$
and neglect radiation losses

\[
\frac{dE_{hs}}{dt} = \frac{\varepsilon_\alpha}{16} \sum_\alpha P^2 V - P \frac{dV}{dt}
\]

\[
\frac{1}{E_{hs}} \frac{dE_{hs}}{dt} = \frac{\varepsilon_\alpha}{24} \sum_\alpha P - 2 \frac{\dot{R}}{R}
\]

\[V = \frac{4\pi}{3} R^3 \Rightarrow dV = 4\pi R^2 dR = 3V \frac{dR}{R} \]

\[E_{hs} = \frac{3}{2} PV = 2\pi PR^3 \]

The hot spot energy increases when

\[\tau_\alpha < \tau_{\text{exp}} \]

\[\text{Ignition condition} \]

\[\text{Alpha heating time} \]

\[\text{Expansion time} \]

\[\text{Ignition condition} \]
Better form of the hot-spot ignition condition. Gbar pressures are needed. Not a lot of energy needed.

\[ \tau_\alpha \sim \frac{24}{\varepsilon_\alpha \sum_\alpha P} < \tau_{\exp} \sim \frac{R}{C_{\text{shell}}} \]

\[ P \tau_{\exp} \sim \frac{PR}{C_{\text{shell}}} > \frac{24}{\varepsilon_\alpha \sum_\alpha} \approx 10 \text{atm} \cdot \text{s} \]

\[ C_{\text{shell}} \sim \sqrt{\frac{5P}{3 \rho}} \Rightarrow P(\text{Gbar}) > 12 \left( \frac{R}{100 \mu m} \right) \left( \frac{1 \text{mg}}{M_{\text{shell}}} \right) \]

\[ E_{\text{hs}} (kJ) = \frac{4\pi}{3} R^3 \left( \frac{3}{2} P \right) > 8 \left( \frac{R}{100 \mu m} \right)^4 \left( \frac{1 \text{mg}}{M_{\text{shell}}} \right) \]

- A bit optimistic since assumes T>6keV and no radiation losses
How can we assemble this optimal configuration with a hot region surrounded by a dense cold shell? By a spherical implosion of a hollow shell.

The solution of these two coupled equations can be singular. That happens at ignition.

\[ E_{hs} = 2\pi PR^3 \]

\[ \frac{1}{E_{hs}} \frac{dE_{hs}}{dt} = \frac{\varepsilon_\alpha}{24} \sum \alpha P - 2 \frac{\dot{R}}{R} \]

\[ M_{sh} \ddot{R} = 4\pi R^2 P \]

Same 2 eqs as before but now R(0) starts negative R(0) = -U_i

Compression when R < 0. Expansion for R > 0
The ignition condition is the same as before but now we know what is the hot spot energy at stagnation

\[ E_{hs}(stag.) = 2\pi P_s R_s^3 = \theta \frac{1}{2} M_{sh} V^2 \]

Hot spot energy at stagnation is a fraction \( \theta \) of the shell kinetic energy

- Substitute into the ignition conditions to find the shell kinetic energy required for ignition

\[ E_{kinetic}^{ignition} \sim 2.7(kJ)\alpha_s^3 \left( \frac{3 \cdot 10^7}{U_i(cm/s)} \right)^7 \]

Where \( \alpha_s \equiv \frac{P_{stag}(\text{Mbar})}{2\rho_{stag}(g/cc)^{5/3}} \) is a measure of the entropy at stagnation and usually referred to as stagnation adiabat

\[ \theta = \frac{V_{hs}}{V_{tot}} = \frac{1}{8} \]
The entropy of the shell (adiabat) at stagnation increases due to the entropy generated by the return shock traveling outwards through the shell.

- Consider the easy planar problem of a supersonic foil hitting a wall, Mach >> 1.

\[ \rho_s = 4 \rho_{if} \]

\[ \alpha_s = 0.22 \alpha_{if} \text{Mach}^2 \]

\[ \frac{3}{2} P_s d = \frac{1}{2} \rho_{if} U^2 (4d) \]

\[ \frac{P_s}{\rho_s^{5/3}} = 0.22 \frac{P_{if}}{\rho_{if}^{5/3}} \text{Mach}^2 \]
Similar results applies to spherically imploding targets but the Mach number scaling is different.

\[
\alpha_s = 2.8 \alpha_{if}^{0.8} \left( \frac{U_i \text{ (cm/s)}}{3 \cdot 10^7} \right)^{0.67} \left( \frac{100}{P(Mbar)} \right)^{0.13}
\]

The reflected shock changes the adiabat.

Spherical result for Mach>>1

\[
\alpha_{if} \equiv \frac{P(Mbar)}{2 \rho (g/cc)^{5/3}}
\]
Finally we have the shell kinetic energy required for ignition in terms of the shell entropy (adiabat), implosion velocity and driver pressure.

\[
E_k > 59 \text{(kJ)} \alpha_{if}^{2.4} \left( \frac{3 \times 10^7}{V_{imp}} \right)^5 \left( \frac{P_{\text{shell}}}{100 \text{ Mb}} \right)^{-0.4}
\]

- Numerical fit from LASNEX (Hermann et al, 2002)

\[
E_k > 50 \text{(kJ)} \alpha_{if}^{1.9} \left( \frac{3 \times 10^7}{V_{imp}} \right)^5 \left( \frac{P_{\text{shell}}}{100 \text{ Mb}} \right)^{-0.77}
\]
Implications of the ignition condition

What is needed for ignition with tens of kJ of kinetic energy:
• Shell velocities larger than a few $10^7\text{cm/s}$ $U_i$
• Lowest possible entropy (adiabat) $\alpha_{if}$
• High driver pressure

- Shell velocities must exceed $2 \times 10^7\text{cm/s}$ or the hot spot temperature is below 6keV. In this case the low fusion reaction rate ($\sim T^4$) and the large radiation losses prevent ignition regardless of the kinetic energy.

- Because of thermal conduction the hot spot temperature is dependent on implosion velocity. The compression work rate must exceed the heat conduction losses. If the implosion velocity is too low the hot never gets hot. Usually $T \sim V$. 
Gain occurs after ignition. In order to obtain high gains, the burn wave trigger by the ignition of the hot spot must propagate through the dense shell surrounding the hot spot.

• If $E_K > E_{\text{ignition}}$, the thermonuclear instability is triggered. As a consequence, a thermal conduction wave followed by a supersonic detonation wave ignite the surrounding fuel. Enhanced alpha particle heating ahead of the conduction wave accelerate the burn.

• It is often reasonable to assume that once the thermonuclear instability is triggered, the entire shell is immediately ignited by the supersonic burn wave.
Density and temperature profiles during ignition and burn wave propagation

temperature (movie)  density (movie)
The thermonuclear gain depends on the implosion velocity, hydrodynamic efficiency of the driver and shell areal density.

\[ G = \frac{E_{\text{fusion}}}{E_{\text{Driver}}} \]

\[ E_{\text{fusion}} = \frac{1}{2} \frac{M_{\text{sh}}}{m_i} \xi_0 \times 17.5 \text{MeV} \]

Shell areal density

\[ \xi_0 = \text{fraction burned}= \frac{\rho R}{\rho R + 7 \text{g/cm}^2} \]

\[ E_{\text{fusion}} = \frac{E_K}{m_i U_i^2} \xi_0 \times 17.5 \text{MeV} \]

\[ \eta_h = \text{hydro efficiency}= \frac{E_K}{E_D} \]

\[ G = 150 \left( \frac{\eta_h}{0.07} \right) \left( \frac{3 \cdot 10^7}{U_i (\text{cm} / \text{s})} \right)^2 \left( \frac{\xi_0}{0.3} \right) \]
The hydro-efficiency depends on how the shell is driven. If the shell is driven by ablating material off the surface causing an effective “rocket,” the efficiency can be easily calculated using simple “rocket science.”

\[
M \frac{dU}{dt} = -4\pi R^2 P_a
\]
Shell Newton’s law

\[
\frac{dM}{dt} = -4\pi R^2 \dot{m}_a
\]
Shell mass decreases due to ablation

\[
P_a = \dot{m}_a U_{\text{exhaust}}
\]
Ablation pressure = Abl. Rate X exhaust vel.
Integrating the rocket equations yields the shell velocity, the shell mass and the hydro efficiency that depends on the ablated mass.

- Assume that driver is on till the shell is about $\frac{1}{2}$ initial radius

\[ U_i = U_{ex} \ln\left(\frac{M_{initial}}{M_{final}}\right) \]
\[ E_{Kinetic} = M_{final} \left[U_{ex} \ln\left(\frac{M_{initial}}{M_{final}}\right)\right]^2 / 2 \]
\[ E_{exhaust} = (M_{initial} - M_{final})\left(\frac{U_{ex}^2}{2} + 3P_{ex} / 2 \rho_{ex}\right) \]
\[ U_{ex}^2 \approx P_{ex} / \rho_{ex} \]

\[ \eta_h = \frac{E_{kinetic}}{E_{exhaust}} = \frac{M_f / M_i \left[\ln(M_f / M_i)\right]^2}{4\left(1 - M_f / M_i\right)} \]

Efficiency is maximum for $M_{final} \sim 0.2M_{initial}$. This is not a good operating point for a DT ablator (Direct Drive) but works for a non-DT ablator (Indirect Drive).
Direct and indirect drive are the two ways of driving a target.

Many aspects the physics are in common between direct and indirect drive. Main differences are in the absorption of the driving radiation (UV versus X-ray).
All the ablation related quantities can be calculated by solving the heat equation in the conduction and coronal region. For direct-drive, assume a local deposition of laser intensity at the critical density.
Equations of steady ablation in the conduction region and corona

\[ p + \rho v^2 = C_0 \]
\[ \rho v = C_2 \]

\[ \left( \frac{5}{2} p + \frac{1}{2} \rho v^2 \right) v - \kappa(T) \frac{dT}{dx} = C_3 \]

- The constant \( C_3 \) changes at the critical surface where the laser energy is absorbed and \( C_3^+ - C_3^- = I_L = \text{laser intensity} \)
- The solution yields the hydro quantities in the conduction region

\[ P_a = 57(I_{15} / \lambda_L)^{2/3} \text{ Mbar} \]

\[ \dot{m}_a = 3.26 \cdot 10^5 (I_{15} / \lambda_L^4)^{1/3} \text{ g / cm}^2 \text{s} \]

\[ T_c = 13.7(I_{15} / \lambda_L^2)^{2/3} \text{ g / cm}^2 \text{s} \]

\[ U_{ex} = C_{ST} = 8.75 \cdot 10^7 \left( I_{15} \lambda_L^2 \right)^{2/3} \text{ cm / s} \]
Another important hydrodynamic parameter: the in-flight aspect ratio $A_{if}$

**Definition:** $A_{if} = \text{Max}(\text{target radius}/\text{target thickness}) = \text{Max}(R/\Delta)$

\[
M \frac{dU}{dt} = -4\pi R^2 P
\]

Thin shell approximation

\[
M = -4\pi \rho \Delta R^2
\]

\[
U \approx \dot{R}
\]

\[
A = \frac{R}{\Delta} = \frac{R\ddot{R}}{P / \rho}
\]

\[
\ddot{R} = g \approx \text{const}
\]

\[
R = R_0 - \frac{1}{2} gt^2
\]

\[
U_i = \left[ gt \right]_{R_0/2} = \sqrt{gR_0}
\]

\[
\left[ R\ddot{R} \right]_{R_0} = gR_0 = U_i^2
\]

$A_{\text{max}} \approx \frac{5}{3} \frac{U_i^2}{C_s^2} = 3.3\text{Mach}^2$
In-flight aspect ratio in terms of implosion parameters.
Fast and low-adiabat shells have larger aspect ratios.

Use $\alpha \sim P/\rho^{5/3}$ to write $\rho \sim (P/\alpha)^{3/5}$ in the Mach number.

Use $P \approx P_a = 57(I_{15} / \lambda_L)^{2/3} \text{Mbar}$

**Our simple model**

$$A_{if} \approx \frac{88}{\alpha_{if}^{0.6}} \left( \frac{U_i (cm / s)}{3 \cdot 10^7} \right)^2 \left( \frac{\lambda_L (\mu m)}{0.35} \right)^{0.18} \frac{1}{I_{15}^{0.27}}$$

**Numerical simulations**

$$A_{if} \approx \frac{60}{\alpha_{if}^{0.6}} \left( \frac{U_i (cm / s)}{3 \cdot 10^7} \right)^2 \left( \frac{\lambda_L (\mu m)}{0.35} \right)^{0.18} \frac{1}{I_{15}^{0.27}}$$
Other results not derived here but they can be derived using the simple hydro in these notes and a little help from the simulations for the coefficients

Shell $\rho R$ at peak compression

$$(\rho R)_{fit}^{max} = \frac{1.3}{\alpha_{if}^{0.55}} \left( \frac{E_L (kJ)}{100} \right)^{0.33} \left( \frac{V_I (cm/s)}{3 \cdot 10^7} \right)^{0.06} \text{g/cm}^2$$

Hydrodynamic efficiency

$$\eta_{fit}^h = \frac{0.049}{I_{15}^{0.25}} \left( \frac{V_I (cm/s)}{3 \cdot 10^7} \right)^{0.75}$$

Hydro-efficiency needs no help from simulations. Both formulas are for laser wavelength of 0.35 mic. Homework problem: derive this hydro-efficiency formula using previous notes.
Almost done. Substitute hydro efficiency and shell areal density to get target gain

From before:

$$G = \frac{E_{fusion}}{E_{Laser}}$$

$$\xi_0 = \text{fraction burned} = \frac{\rho R}{\rho R + 7 \text{g/cm}^2}$$

$$G = 150 \left( \frac{\eta_h}{0.07} \right) \left( \frac{3 \cdot 10^7}{U_i (\text{cm/s})} \right)^2 \left( \frac{\xi_0 (\rho R)}{0.3} \right)$$

$$G = 110 \left( \frac{3 \cdot 10^7}{U_i (\text{cm/s})} \right)^{1.25} \left( \frac{\xi_0 (\rho R)}{0.3} \right)$$

$$\frac{(\rho R)_{fit}^{max} = \frac{1.3}{\alpha_{if}^{0.55}} \left( \frac{E_L (kJ)}{100} \right)^{0.33} \left( \frac{V_I (\text{cm/s})}{3 \cdot 10^7} \right)^{0.06}}$$
RECAP of the 1-D WORLD OF INERTIAL FUSION (laser DD)

- Let’s implode a spherical shell, with a laser direct-drive with wavelength of 0.35mic providing 100 Mbar of driving pressure.

- What is the adiabat?  
  \[ \alpha = \frac{P(Mbar)}{2\rho(g/cc)^{5/3}} \]

- How much energy on target is needed to ignite?  
  \[ E_L(kJ) = 10^3 \alpha_{if}^{1.9} \left( \frac{3 \cdot 10^7}{U_i(cm/s)} \right)^{6.65} \]

What is the areal density at peak compression?

\[ (\rho R)_{fit}^{max} = \frac{1.3}{\alpha_{if}^{0.55}} \left( \frac{E_L(kJ)}{100} \right)^{0.33} \left( \frac{V_I(cm/s)}{3 \cdot 10^7} \right)^{0.06} \]
How thick is the shell (relative thickness $\rightarrow$ Aspect ratio) ?

$$A_{if} \approx \frac{60}{\alpha_{if}^{0.6}} \left( \frac{U_i (cm/s)}{3 \cdot 10^7} \right)^2 \frac{1}{I_{15}^{0.27}}$$

What is the laser intensity needed ?

$$P_a = 57(I_{15} / \lambda_L)^{2/3} Mbar \Rightarrow I \approx 1.5 \cdot 10^{15} W / cm^2$$

• What is the energy gain?

$$G = 110 \left( \frac{3 \cdot 10^7}{U_i (cm/s)} \right)^{1.25} \left( \frac{\xi_0 (\rho R)}{0.3} \right) \quad \xi_0 = \frac{\rho R}{\rho R + 7 g/cm^2}$$

$$(\rho R)_{max}^{fit} = \frac{1.3}{\alpha_i^{0.55}} \left( \frac{E_L (kJ)}{100} \right)^{0.33} \left( \frac{V_i (cm/s)}{3 \cdot 10^7} \right)^{0.06}$$
Let’s get some numbers (use LLE point design for NIF DD)

\[ E_L = 1500kJ \]

\[ U_i = 4.3 \cdot 10^7 \text{ cm/s} \]

\[ \alpha_{if} = 2.7 \]

\[ I = 10^{15} \text{ W/cm}^2 \]

- Do we have enough energy to ignite?
  \[ E_{\text{ignition}} = 610kJ \quad \text{Plenty!} \]

- What is the peak areal density?
  \[ (\rho R)_{\text{max}} = 1.9 \text{ g/cm}^2 \]

- What is the in-flight aspect ratio?
  \[ A_{if} = 69 \]

- What is the energy gain?  \( G = 49 \)